

Simple Mechanism

Link / Kinematic link / Element

Each part of machine which moves another one is simply known as kinematic link.

It is also called as elements.

Types of link

i) A rigid link is one which does not undergo any deformation while transmatic motion.
ex:- crank and connecting rod

ii) A flexible link is one which does not partly deform in a manner not to effect the transmission of motion.
ex: belt, ropes, chains.

iii) A fluid link is one which is formed by having a fluid ink in a receptacle and the motion is transmitted through fluid by pressure of compression.
ex: hydraulic presses, jacks and brakes.

Structure

It is an assemblage of a numbers of resistant body having no relative motion between them. e.g :- railway bridge, roof truss, machine frames

Kinematic Pair

- The two links are element of a machine when in contact with are said to form a pair.
- If the relative motion between them is completely or successfully constrained. The pair is known as kinematic pair.

Classifications of kinematic pair
(According to the type of relative motion between the elements)

- i) Sliding pair
- ii) Turning pair
- iii) Rolling pair
- iv) Screw pair
- v) Spherical pair

(According to the type of contact between the elements)

(a) Lower pair

When two elements of pair have a surface contact between them is known as lower pair. Here one element slides over another.

(b) Higher Pair

When two elements of pair have a point or line contact that is known as higher pairs.

Chain

Where the kinematic pairs are coupled in such a way that the first link is joined to the last link to transmit definite motion. It is called as kinematic chain.

$$l = 2p - 4$$

Mechanism

When one of the member (link) is of a kinematic chain is fixed that is known as mechanism.

Kinematic Chain

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion, it is called a kinematic chain.

Machine

When a mechanism is required to transmit power or to do some particular type of work, it becomes a machine.

Q.1

L - links - 3

J - Joints - 3

P - Pair - 3

$$C = 2P - 4$$

We know $L = 3$

$$= 2 \times 3 - 4$$

$$= 2$$

$\therefore$ L.H.S $>$ R.H.S.

$$J = \frac{3}{2} \times 1 - 2$$

$$= \frac{9}{2} - 2$$

$$= 4.5 - 2$$

$$= 2.5$$

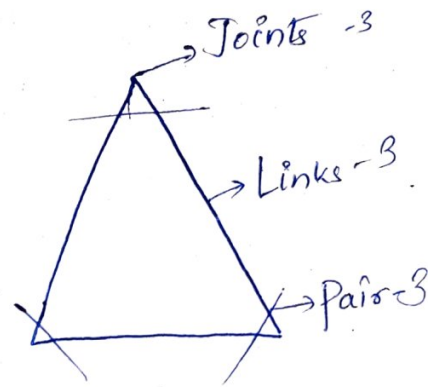
We know $J = 3$

$\therefore$ L.H.S $>$ R.H.S.

Not a kinematic Chain

$$L.H.S > R.H.S$$

$\rightarrow$ Locked chain



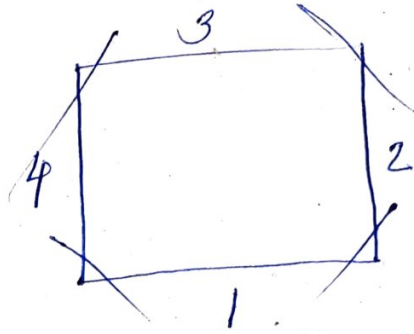
Q.

Q.2

Joints - 04

Links - 04

Pair - 04



$$\begin{aligned}\text{links } (L) &= 2P - 4 \\ &= 2 \times 4 - 4 \\ &= 4\end{aligned}$$

$$\therefore L = 4$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$\begin{aligned}\text{Joints } (J) &= \frac{3}{2}L - 2 \\ &= \frac{3}{2} \times 4 - 2 \\ &= 6 - 2 \\ &= 4\end{aligned}$$

$$\therefore J = 4$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

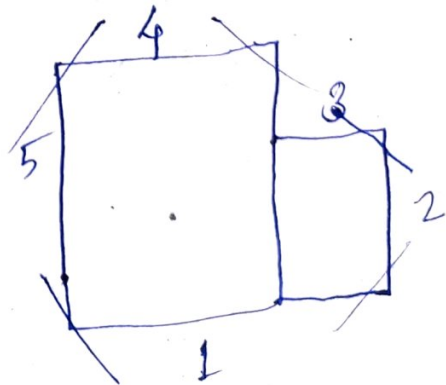
It is a kinematic Chain

Q.3

$$L = 6$$

$$J = 7$$

$$P = 5$$



We know that $l = 2P - 4$
 $= 2 \times 5 - 4$
 $= 6$

$\therefore L = 6$

$\therefore L.H.S = R.H.S.$

We know that $(J) = \frac{3}{2}l - 2$
 $= \frac{3}{2} \times 6 - 2$
 $= 9 - 2$
 $= 7$

$\therefore J = 7$

$\therefore L.H.S = R.H.S$

It is a kinematic chain

Q.5

links $(l) = 10$

Points $(J) = 1 + (6 \times 2)$

$J = 13$

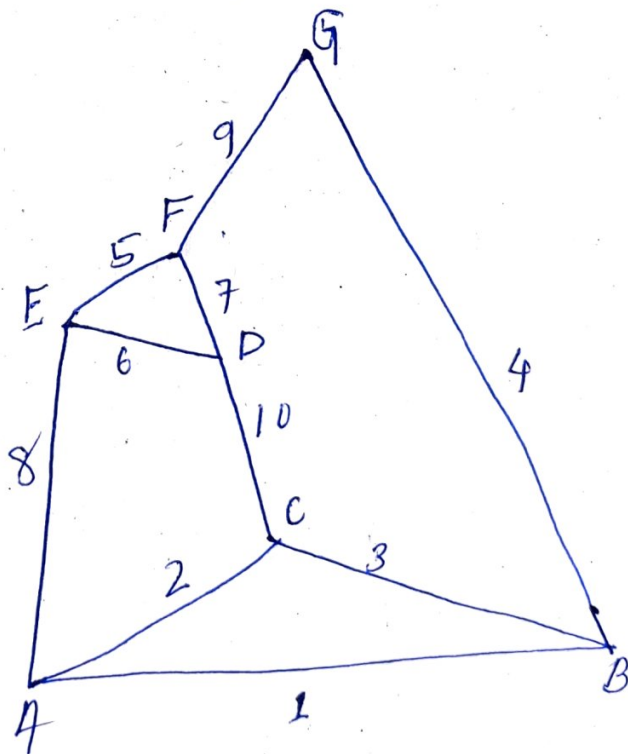
We know that

$J = \frac{3}{2}l - 2$

$= \frac{3}{2} \times 10 - 2$

$= 15 - 2 = 13 \quad \therefore L.H.S = R.H.S$

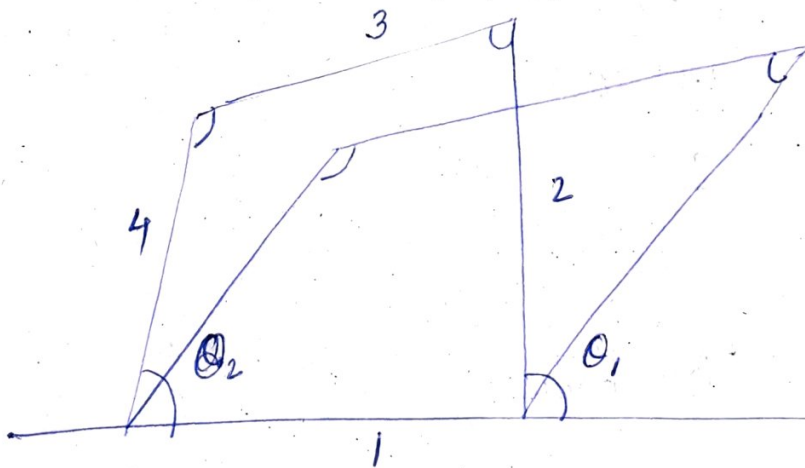
It is a kinetic chain.



DT: 29th Jan 20

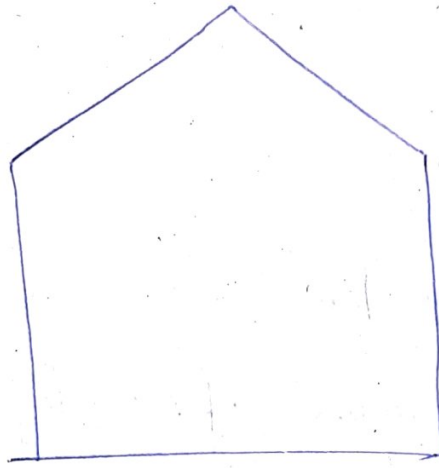
Degree of freedom

The degree of freedom of a mechanical system is the number of ~~configuration~~ independent parameter that defines its configuration.



$$n = 3(l-1) - 2J - h$$

$$n = 1$$



$$\begin{aligned} n &= 3(l-1) - 2J - h \\ &= 3(5-1) - 2 \times 5 - 0 \\ &= 3 \times 4 - 10 - 0 \\ &= 12 - 10 \\ &= 2 \end{aligned}$$

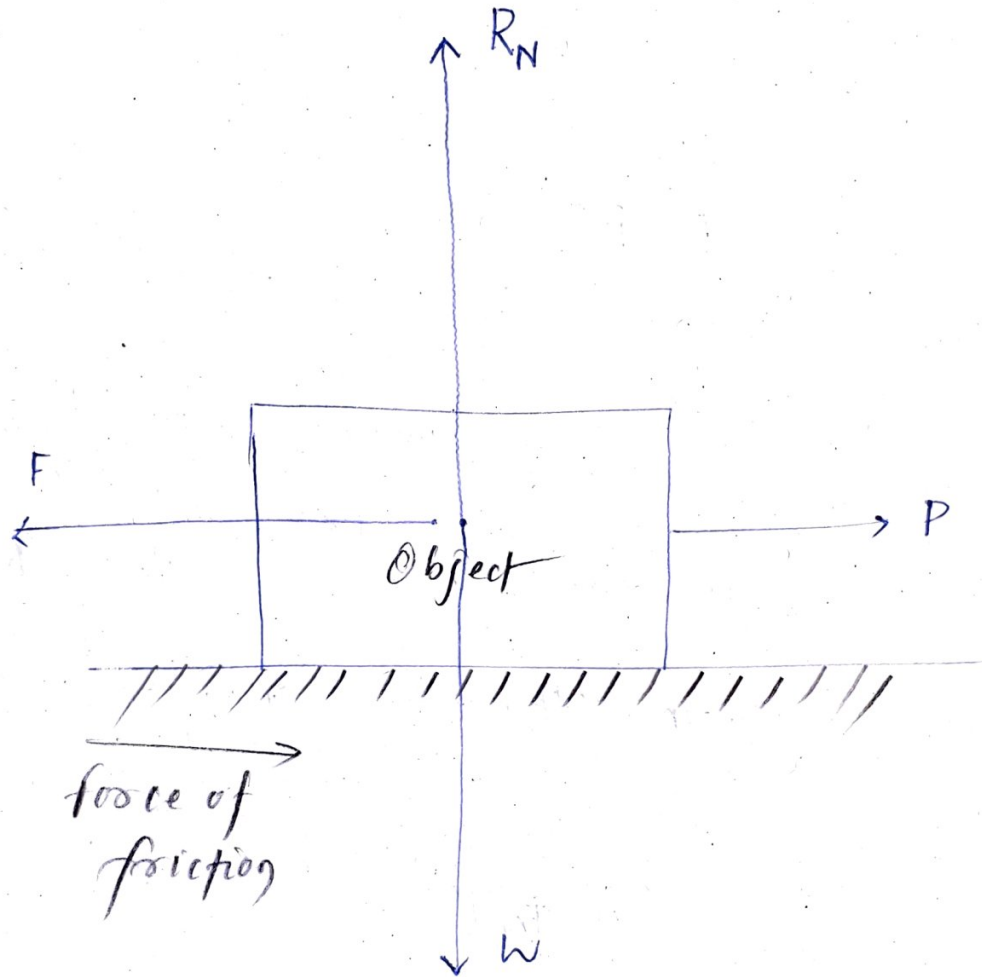
Inversion Mechanism

- By too obtain many mechanism as the number of links in a kinematic chain by fixing, in turn different links in a kinematic chain. This method of obtaining

different mechanisms by fixing different link in a kinematic chain is known as inversion of a mechanism.

1

Friction



Laws Of friction

The force of friction always acts in a direction always opposite to that the body tends to move.

$$\vec{P} = -\vec{F}$$

The magnitude of force of friction is exactly equals to the force which tends the body to move.

$$F = P$$

The magnitude of limiting friction bears a constant ratio to the normal reaction between the two surface.

$$\frac{F}{R_N} = \text{constant}$$

The force of friction is independent of the area of contact between two surface.

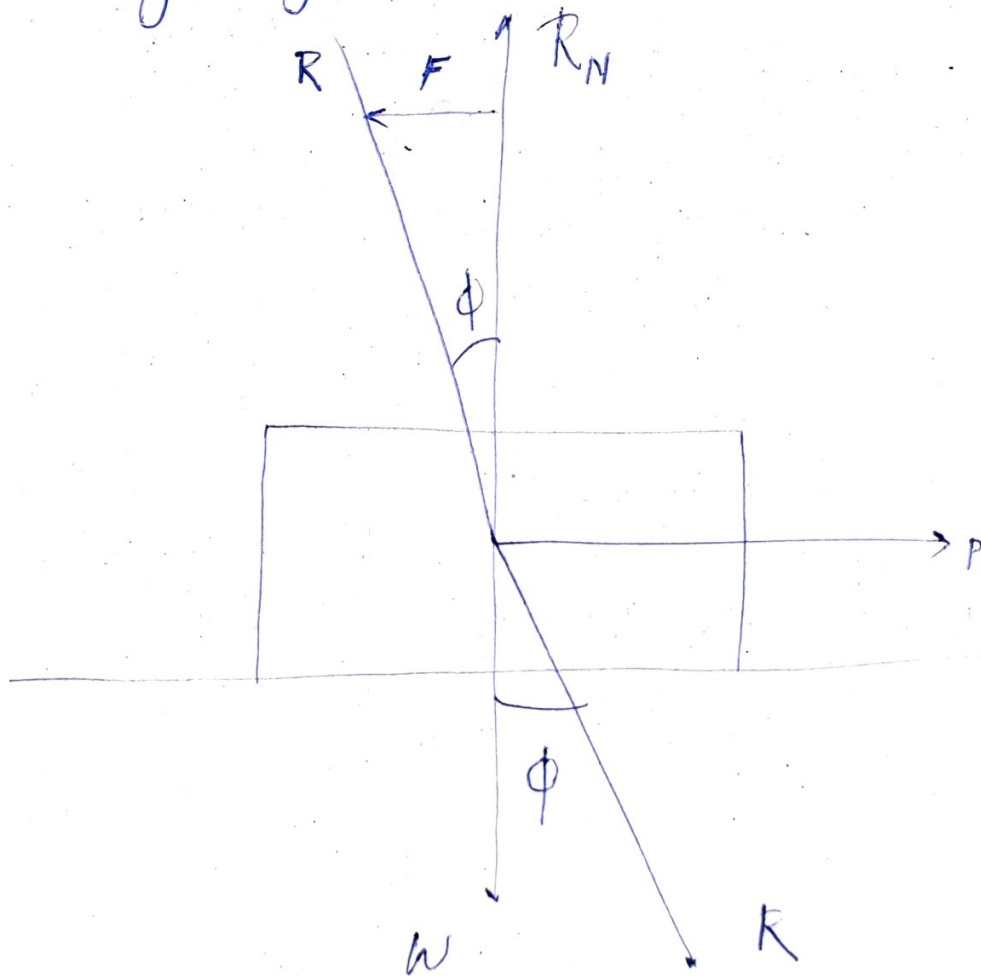
The force of friction is depends upon the roughness of the surface.

Coefficient of friction

It is defined as the ratio of limiting friction to the normal reaction between two bodies. mathematically coefficient of friction is

$$\mu = \frac{F}{R_N}$$

limiting angle of friction



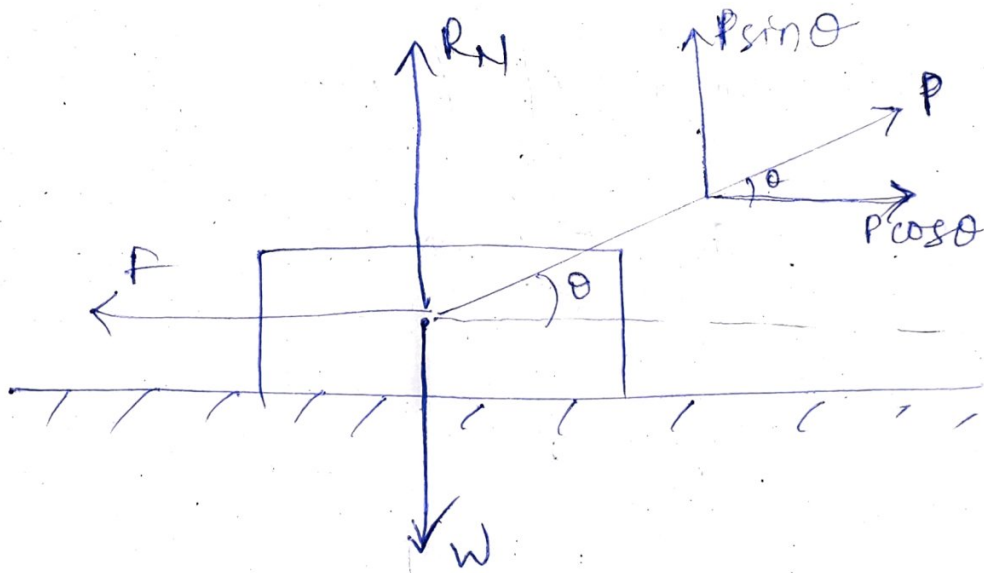
$$\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{F}{R_N} = \frac{\mu R_N}{R_N}$$

$$\tan \phi = \mu$$

$$\Rightarrow \phi = \mu$$

(Neglecting Small component)

Force required to pull a body



$$\begin{array}{l|l} \checkmark W = R_N + P \sin \theta & \checkmark F = P \cos \theta \\ \Rightarrow R_N = W - P \sin \theta & \end{array}$$

F = friction

$$F = \mu R_N$$

$$= \mu (W - P \sin \theta)$$

$$f = \tan \phi (w - P \sin \theta)$$

$$= \frac{\sin \phi}{\cos \phi} (w - P \sin \theta)$$

$$\textcircled{P} P \cos \theta \cdot \cos \phi = \sin \phi (w - P \sin \theta)$$

$$\Rightarrow P \cos \theta \cdot \cos \phi = w \sin \phi - P \sin \theta \cdot \sin \phi$$

$$\Rightarrow P \cos \theta \cdot \cos \phi + P \sin \theta \cdot \sin \phi = w \sin \phi$$

$$= P \cos(\theta - \phi) = w \sin \phi$$

$$\Rightarrow \boxed{P = \frac{w \sin \phi}{\cos(\theta - \phi)}}$$

Friction

Friction is a force that opposes the motion of two contacting surfaces.

Types of Friction

1. Static Friction
2. Dynamic Friction

Static Friction

It is the friction, experienced by a body when at rest.

Dynamic Friction

It is the friction, experienced by a body when in motion. It is also called

kinetic friction.

It is three types:

- a) Sliding friction
- b) Rolling friction
- c) Pivot friction

Sliding Friction

It is the friction, experienced by a body,

when it slides over another body.

Rolling friction

It is the friction, experienced between the surfaces which has balls or rollers interposed between them.

Pivot friction

It is the friction, experienced by a body, due to the motion of ~~friction~~ rotation as in case of foot step bearings.

Further be classified as:

1. Friction between unlubricated surface.
2. Friction between lubricated surface.

Friction between unlubricated surface

The friction experienced between two dry and unlubricated surfaces in contact is known as dry or solid friction.

Friction between lubricated surfaces

Boundary
friction

It is the friction, experienced between the rubbing surfaces, when the surfaces have a very thin

layer of lubricant.

Viscosity

~~It is a measure of the res~~

Fluid friction

It is the friction, ~~between~~ experienced between the rubbing surfaces, when the surfaces have a thick layer of lubricant.

Limiting friction

The maximum value of force of friction, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting friction.

Laws of Static Friction

1. The force of friction always acts in the direction opposite to that in which the body tends to move.

2. The magnitude of force of friction

is exactly equal to the force, which tends the body to move.

3. The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces.

$$\boxed{\frac{F}{R_N} = \text{constant}}$$

4. The force of friction is independent of the area of contact between the two surfaces.
5. The force of friction depends upon the roughness of the ~~and~~ surface.

Laws of Kinetic or Dynamic friction

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But the ratio is slightly less than in case of

limiting friction.

3. For ~~the~~ moderate speeds, the force of friction remains constant. But it decreases slightly with the increase speed.

Laws of Solid friction

1. The force of friction is directly proportional to the normal load between the surfaces.
2. The force of friction is independent of the area of the contact surface for a given normal ~~load~~ load.
3. The force of friction depends upon the material of which the contact surface are made.
4. The force of friction is independent of the velocity of sliding of one body relative in the other body.

Laws of fluid friction

1. The force of friction is almost independent of the load.
2. The force of friction reduces with the temperature of the lubricant.
3. The force of friction is independent of the substances of bearing surfaces.
4. The force of ~~is~~ of friction is different for different lubricants.

Coefficient of friction

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies.

$$\mu = F/R_N$$

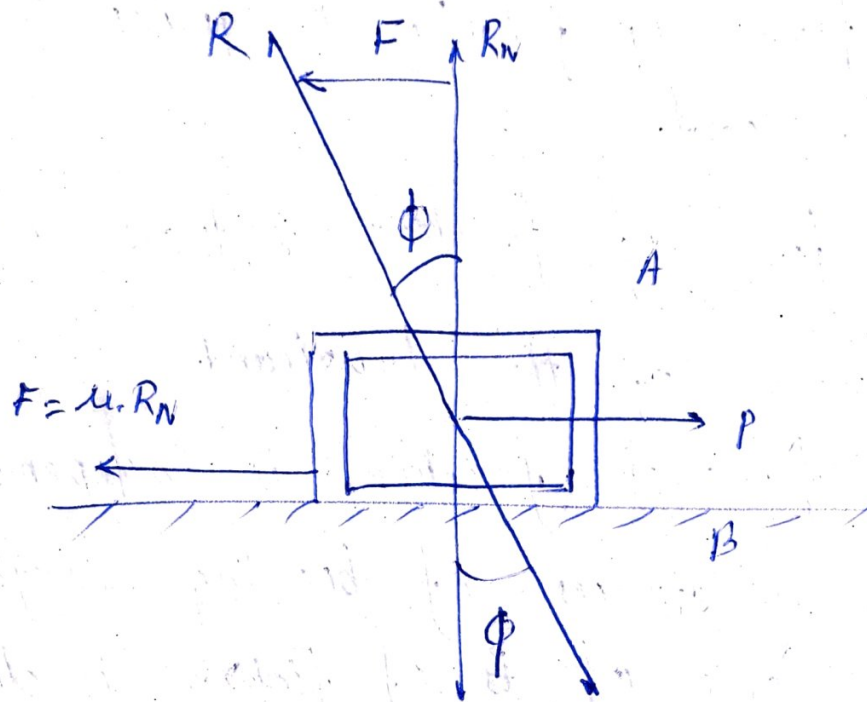
Limiting Angle of friction

Consider a body of weight = W

horizontal force = P

force of friction = F

coefficient of friction = μ



Reaction force = R_N

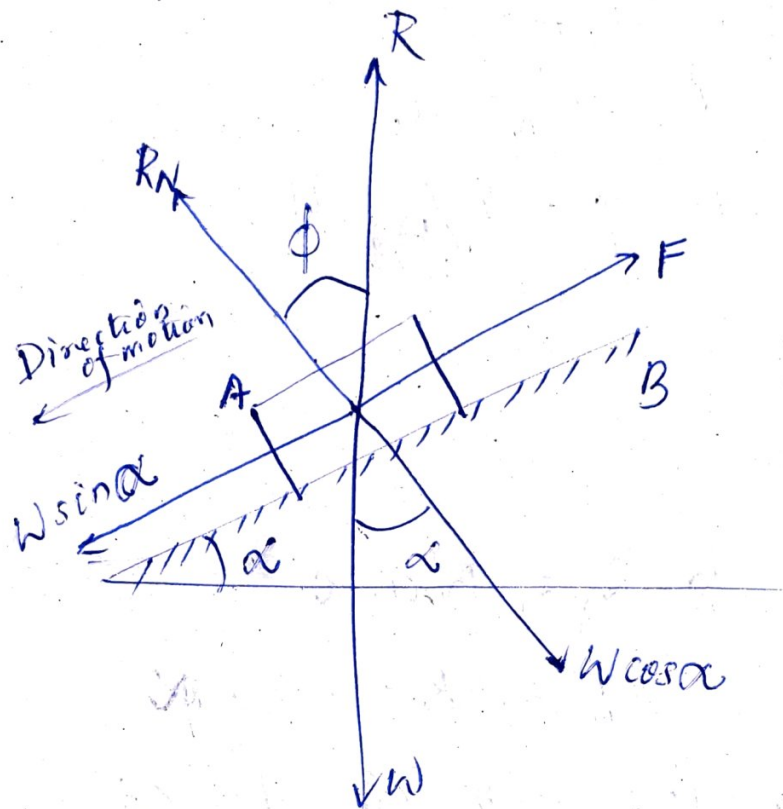
ϕ = limiting angle of friction

$$\tan \phi = \frac{F}{R_N}$$

$$= \frac{\mu R_N}{R_N}$$

$$\tan \phi = \mu$$

Angle of repose



W = Weight of the body

$R = \frac{W \cos \alpha}{\mu}$

R_N = Normal reaction

$F = \mu R_N$ = force of friction

If the angle of inclination ' α ' of the plane to the horizontal is such that the body begins to move down the plane, then the angle α is called the angle of repose.

We know that

$$F = \mu R_N = \mu \cdot W$$

$$\text{and } \tan \phi = \mu \quad \text{--- (i)}$$

Here,

$$R_N = W \cos \alpha \quad \text{--- (ii)}$$

$$F = \mu R = W \sin \alpha \quad \text{--- (iii)}$$

dividing eqⁿ (iii) by (ii)

we get

$$\frac{\mu R}{R} = \frac{W \sin \alpha}{W \cos \alpha}$$

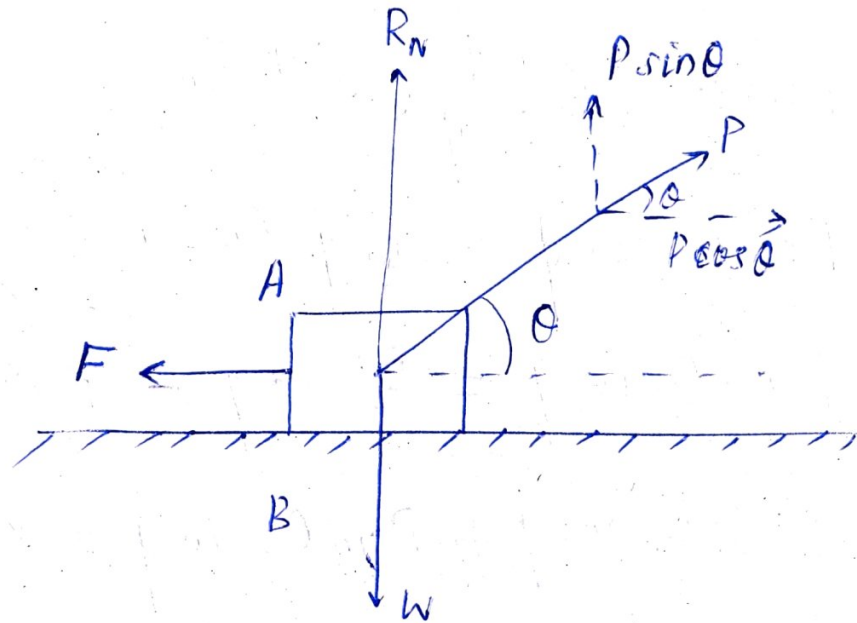
$$\Rightarrow \mu = \tan \alpha \quad \text{--- (iv)}$$

From equation (iv) & (i)

$$\tan \alpha = \mu = \tan \phi$$

$$\Rightarrow \alpha = \phi$$

Minimum force required to slide a body on a rough horizontal plane



Here,

$$W = R_N + P \sin \theta$$

$$\Rightarrow R_N = W - P \sin \theta \quad \text{--- (i)}$$

~~and~~

and

$$P \cos \theta = F = \mu \cdot R_N \quad \text{--- (ii)}$$

Substituting the value of R_N from equation (i),

$$P \cos \theta = \mu (W - P \sin \theta)$$

④

$$= \tan \phi (W - P \sin \theta)$$

$$= \frac{\sin \phi}{\cos \phi} (W - P \sin \theta)$$

$$\Rightarrow P \cos \theta = \frac{\sin \phi}{\cos \phi} (W - P \sin \theta)$$

$$\Rightarrow P \cos \theta \cdot \cos \phi = \sin \phi (W - P \sin \theta)$$

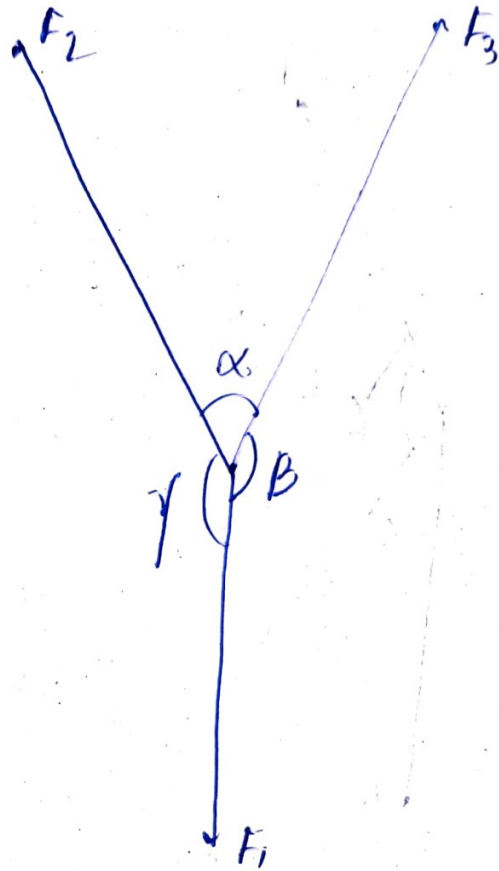
$$\Rightarrow P \cos \theta \cdot \cos \phi = W \sin \phi - P \sin \theta \cdot \sin \phi$$

$$\Rightarrow P \cos \theta \cdot \cos \phi + P \sin \theta \cdot \sin \phi = W \sin \phi$$

$$\Rightarrow P \cos(\theta - \phi) = W \sin \phi$$

$$\Rightarrow \boxed{P = \frac{W \sin \phi}{\cos(\theta - \phi)}}$$

Lami's Theorem

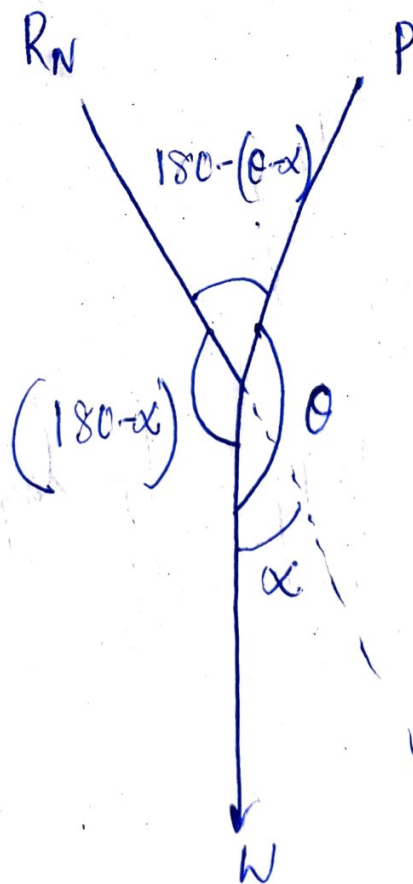
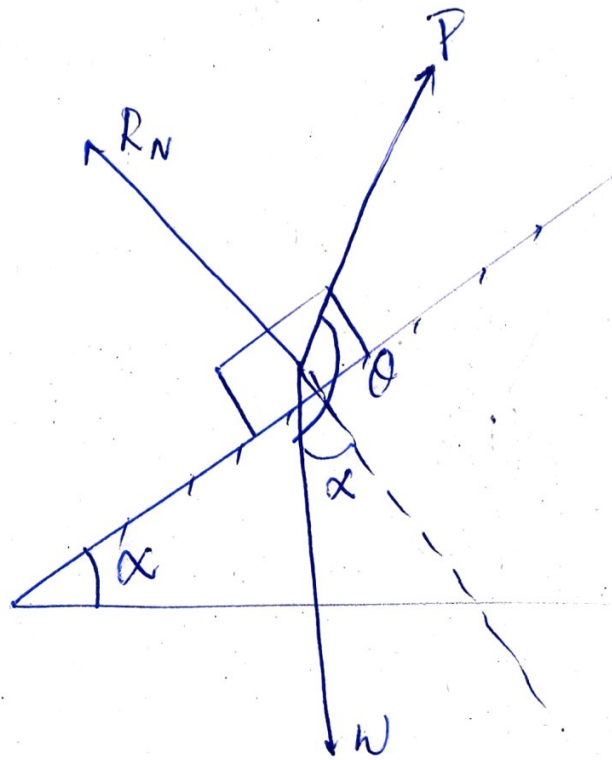


$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

Friction of a body by laying on
a rough inclined plane

i) Considering the motion of the body
up the plane.

b)



$$\frac{W}{\sin(180-(\theta-\alpha))} = \frac{P}{\sin(180-\alpha)} = \frac{R}{\sin \theta}$$

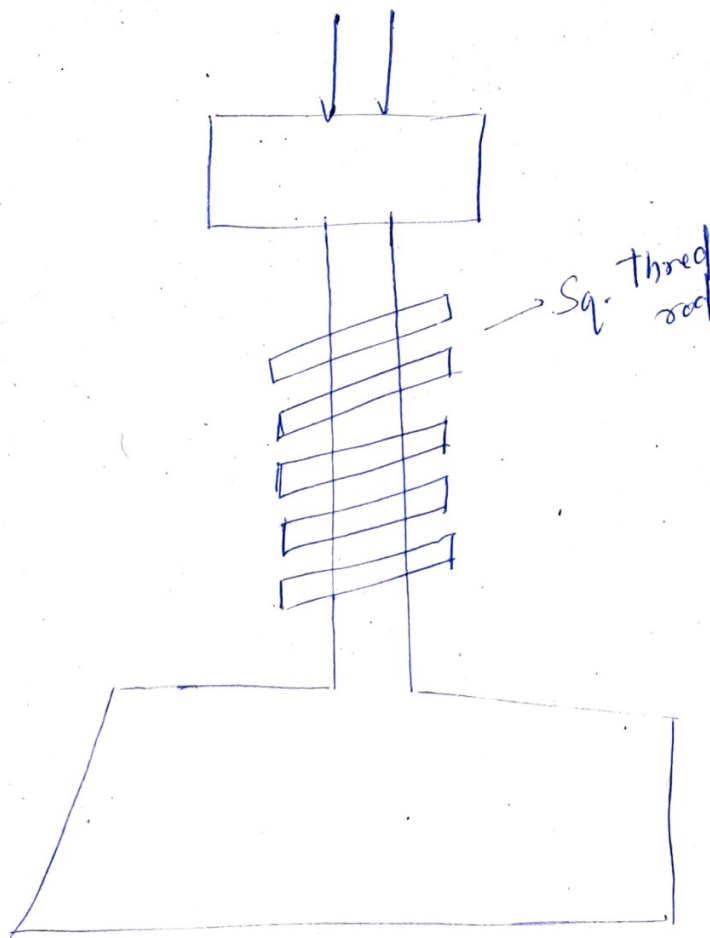
$$\Rightarrow \frac{W}{\sin(\theta-\alpha)} = \frac{P}{\sin \alpha}$$

$$\Rightarrow \cancel{W \sin \alpha} P = \frac{W \sin \alpha}{\sin(\theta-\alpha)}$$

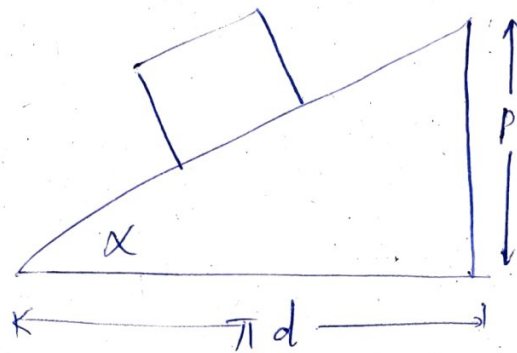
Screw Jack

A screw jack is a device for lifting heavy load by comparatively applying a smaller effort at its handle.

The principle on which screw jack work is similar to that of an inclined plane.



Torque required to lift the load by
screw jack



P is diameter of the screw.

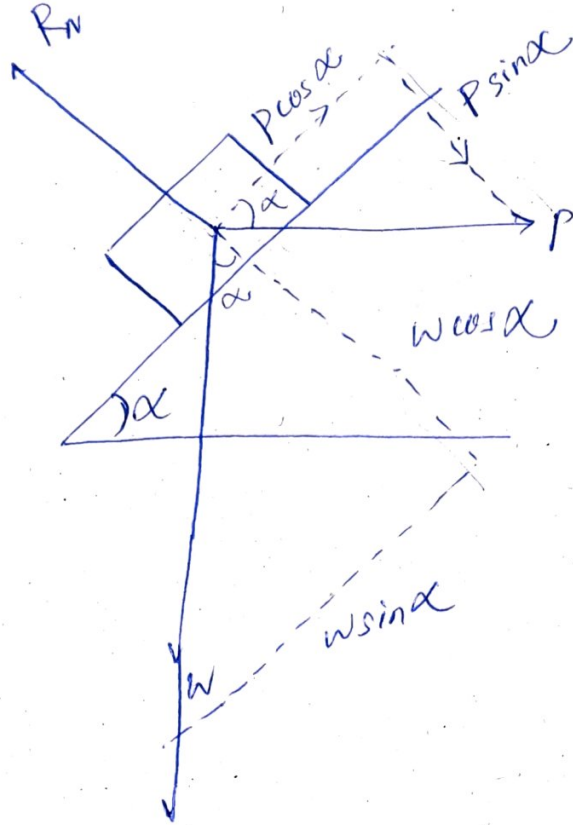
d is the mean diameter of the screw

α is helix angle.

P is effort applied

W is load to be lifted.

μ = co-efficient of friction



Resolving along the plane

Horizontally

$$P \cos \alpha = W \sin \alpha + F$$

$$= W \sin \alpha + \mu R_N$$

Vertically

$$R_N = P \sin \alpha + W \cos \alpha$$

$$\Rightarrow P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$

$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

$$\Rightarrow P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

$$\Rightarrow P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$$P = W \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}$$

$$= W \left(\frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha} \right)$$

$$= W \left(\frac{\sin \alpha + \frac{\sin \phi}{\cos \phi} \cos \alpha}{\cos \alpha - \frac{\sin \phi}{\cos \phi} \sin \alpha} \right)$$

$$= W \frac{\sin \alpha \cdot \cancel{\cos \phi} + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \phi \sin \alpha}$$

$$= W \frac{\sin \alpha + \phi}{\cos \alpha + \phi}$$

$$P = W \tan(\alpha + \phi)$$

Torque required to overcome the friction between the screw & nut.

Practice Twice

Q// A electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm per ~~high~~ minute.

The screw has a single square thread of 6 mm pitch. on a major diameter of 40 mm. Co-efficient of friction at the screw thread is 0.1. Estimate the power of motor.

$$W = 75 \text{ kN}$$

$$P = 6 \text{ mm}$$

$$\text{rotation} = 300 \text{ mm per minute.}$$

$$\mu = 0.1$$

$$\text{Major diameter} = 40 \text{ mm} \\ (d)$$

We know that-

$$P = W \tan (\alpha + \phi)$$

$$= W \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \cdot \tan \phi}$$

$$\tan \alpha = \frac{P}{\pi d} = 6$$

$$d = \phi_0 - P/2$$

$$= 40 - 6/2$$

$$= 37 \text{ mm}$$

$$\tan \alpha$$

$$= \frac{P}{\pi d}$$

$$= \frac{6}{\pi \times 37}$$

$$= 0.051$$

$$P = \frac{\omega \tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}$$

$$= 75 \times 10^3 \left(\frac{0.051 + 0.1}{1 - 0.051 \times 0.1} \right)$$

$$= 11383.05 \text{ N}$$

12/02/20

$$T = P \times d/2$$

$$\text{Work done} = 2\pi N X T$$

$$= W \tan(\alpha + \phi) d/2$$

Q// A turnbuckle with righthand left hand ~~on~~ screw used to couple a wagon. The thread Pitch is 12mm and mean diameter 40mm. Co-efficient of friction $\mu = 0.16$. Determine the workdone. In drawing the wagon together a distance of 240mm against a steady load of 2500 N.

$$\text{Pitch} = 12 \text{ mm}$$

$$d = 40 \text{ mm}$$

$$\mu = 0.16$$

$$W = 2500 \text{ N}$$

$$\tan \alpha = \frac{P}{\pi d}$$

$$= \frac{12}{\pi \times 40} = 0.095$$

$$T = P \times d/2$$

$$P = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \cdot \tan \phi} \right]$$

$$= 2500 \left[\frac{0.095 + 0.16}{1 - 0.095 \times 0.16} \right]$$

$$= 647.33 \text{ N}$$

$$T = P \times d/2$$

$$= 647.33 \times \frac{40}{2}$$

$$= 12946.791$$

$$= 12946.8 \text{ N.mm}$$

$$N = 10$$

$$W = 2\pi \times 10 \times 12946.8$$

$$= 813.47 \text{ N.m}$$

Q. A 150 mm diameter bolt against which a steam pressure of 2 MN/m^2 is acting is closed by means of a square threaded screw. 50 mm in external diameter with 6 mm pitch. If coefficient of friction is 0.12 find the torque required to turn the handle.

$$D = 150 \text{ mm} = 0.15 \text{ m}$$

$$W = 2 \times 10^6 \times \frac{\pi}{4} \times (0.15)^2$$

$$= 35342.91 \text{ N}$$

$$d = d_o - P/2$$

$$= 50 - 6/2$$

$$= 47 \text{ mm} =$$

$$\mu = 0.12 = \tan \phi$$

$$\tan \alpha = \frac{P}{\pi d} = \frac{0.006}{\pi \times 0.047}$$

$$= 0.0406$$

$$P = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \cdot \tan \phi} \right]$$

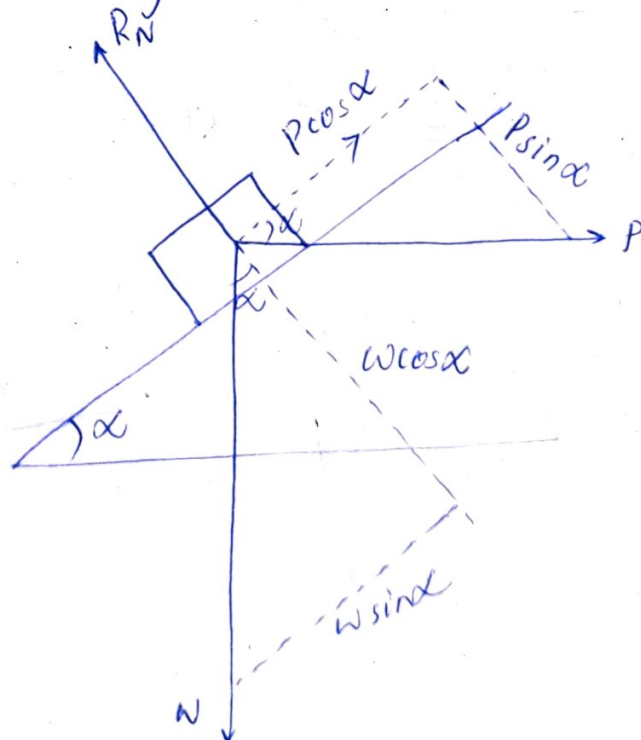
$$= 35342.91 \times \left[\frac{40.63 + 0.12}{1 - 40.63 \times 0.12} \right] \times$$

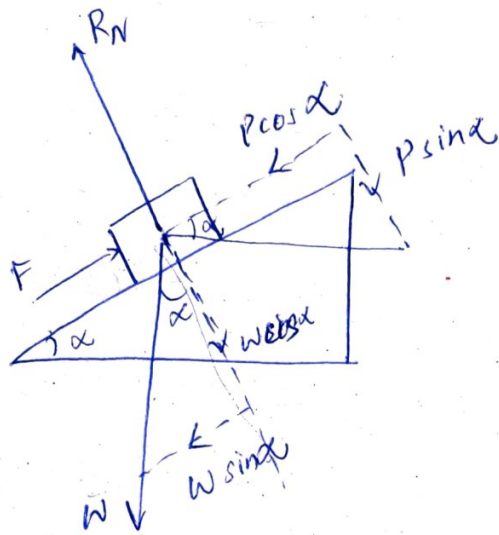
$$= \cancel{35342.91}$$

$$35342.91 \times \left[\frac{0.0406 + 0.12}{1 - 0.0406 \times 0.12} \right]$$

$$= 5703.86 \text{ N/m}$$

~~Q~~ Torque required to lower a body
by a screw jack





$$P \cos \alpha + W \sin \alpha = F$$

$$\Rightarrow P \cos \alpha = F - W \sin \alpha$$

$$= \mu R_N - W \sin \alpha$$

$$R_N = W \cos \alpha + P \sin \alpha$$

①

$$P \cos \alpha = \mu R_N - W \sin \alpha$$

$$= \mu (W \sin \alpha + P \sin \alpha) - W \sin \alpha$$

$$= \mu W \sin \alpha + \mu P \sin \alpha - W \sin \alpha$$

$$\Rightarrow P \cos \alpha - \mu P \sin \alpha = \mu W \sin \alpha - W \sin \alpha$$

$$\Rightarrow \cancel{P \cos \alpha} - \cancel{\mu P \sin \alpha} \quad P(\cos \alpha - \mu \sin \alpha) = W(\mu \sin \alpha - \cos \alpha)$$

Q1/

$$d = 50 \text{ mm}$$

$$p = 10 \text{ mm} =$$

$$\mu = 0.15 = \tan \phi$$

$$l = 0.7 \text{ m}$$

$$W = 20 \text{ kN}$$

$$\tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

Upward

$$p = W \tan(\alpha + \phi)$$

$$= 20 \times 10^3 \times \left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15} \right]$$

$$= 4315.23 \text{ N/mm}$$

Downward

$$P = W \left[\frac{\tan \phi - \tan \alpha}{1 + \tan \phi \times \tan \alpha} \right]$$

$$= 20 \times 10^3 \left[\frac{\cancel{0.0637} \quad 0.15 - 0.0637}{1 + 0.15 \times 0.0637} \right]$$

$$= 1709.66 \text{ N}$$

$$= 17010 \text{ N}$$

Dt: 18/02/20

Efficiency of the screw jack

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

Q// The pitch of 50 mm mean diameter threaded screw of a screw jack is 12.5 mm. The coefficient of friction is 0.13. Determine the torque required to rise the load of 25 kN. Assuming the load to rotate the with the screw. Determine the ratio of torque.

Data given,

$$d = 50 \text{ mm}$$

$$p = 12.5 \text{ mm}$$

$$\tan \phi = 0.13$$

$$W = 25 \text{ kN}$$

$$\tan \alpha = \frac{p}{\pi d} = \frac{12.5}{\pi \times 50} = 0.079$$

$$P = W \tan(\alpha + \phi)$$

$$= \frac{W \tan \alpha + \tan \phi}{1 - \tan \alpha \cdot \tan \phi}$$

$$= 25 \times 10^3 \left(\frac{0.079 + 0.13}{1 - 0.079 \times 0.13} \right)$$

$$= 5279.21 \text{ N}$$

$$\text{Torque} = P \times \frac{d}{2}$$

$$= \cancel{5279} 5279 \times \frac{50}{2}$$

$$= 5279.21 \times \frac{50}{2}$$

$$= 131980.25 \text{ N.mm}$$

Efficiency,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

$$= \frac{\tan \alpha}{\tan \alpha + \tan \phi}$$

$$= \frac{\tan \alpha}{1 - \tan \alpha \tan \phi}$$

$$= \frac{0.079}{0.211}$$

$$= 0.374$$

$$= 37.4\%$$



7



Bearing

Flat pivot Bearing

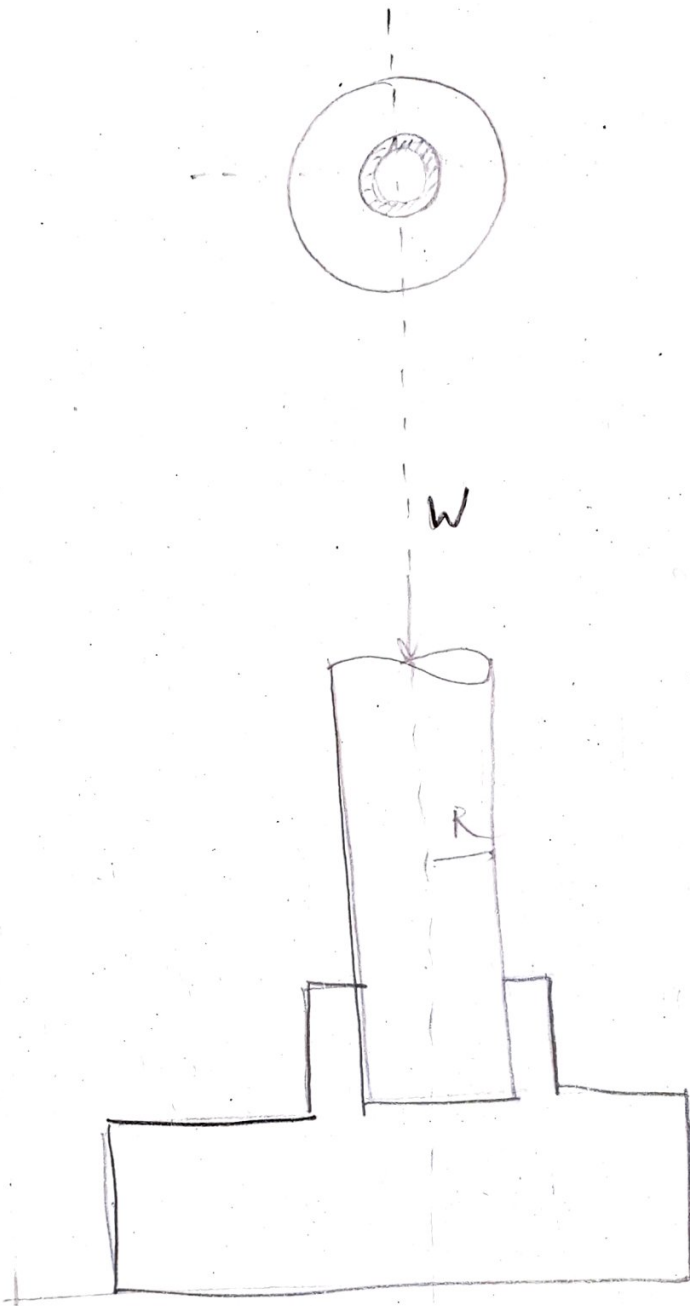
When shaft rotates in a flat pivot bearing the sliding friction will be along the surface of contact between the shaft and bearing.

W is the load transmitted

R is the radius of bearing surface

p is the intensity of pressure per unit area.

μ is the coefficient of friction.



for finding torque transmitted we
consider two cases uniform pressure
i) Uniform Pressure

For uniformly distributed

$$\text{pressure} = \frac{W}{A}$$

$$= \frac{W}{\pi R^2}$$

Considering a small layer of radius 'r'
and thickness 'dr'

Area of the layer,

$$A = 2\pi r \cdot dr$$

Then load transmitted by the area

$$\delta W = P \times A$$

$$\delta W = P \times 2\pi r \cdot dr$$

Frictional Resistance by the layer

Friction by layer

$$F_r = \mu \delta W$$

$$= \mu P 2\pi r \cdot dr$$

Torque Transmitted by layer

$$T_r = F_r \times r$$
$$= \mu P 2\pi r^2 dr$$

Total Torque

$$T = \int_0^R T_r$$

$$= \int_0^R \mu P 2\pi r^2 dr$$

$$= \mu P 2\pi \int_0^R r^2 dr$$

$$= \mu P 2\pi \left[\frac{r^3}{3} \right]_0^R$$

$$= \mu P 2\pi \left[\frac{R^3}{3} - \frac{0^3}{3} \right]$$

$$= \mu P 2\pi \frac{R^3}{3}$$

$$T = \frac{2}{3} \mu P \pi R^3$$

$$= \frac{2}{3} \mu \frac{W}{\cancel{\pi R^2}} R^3$$

$$T = \frac{2}{3} \mu W R$$

Considering Uniform Wear

The rate of wear is proportional to the product of intensity of pressure and the velocity of rubbing surface.

Since the velocity of rubbing surface increase with increase with distance therefore from the axis of bearing therefore for uniform wear

$$C = \text{constant}$$

$$P = \frac{C}{r}$$

$$\begin{aligned} \delta W &= P \times A \\ &= \frac{C}{r} \times 2\pi r \times d \times \delta r = 2\pi C d \delta r \end{aligned}$$

$$W = \int_0^R \delta W$$

$$= \int_0^R 2\pi c dr$$

$$= 2\pi c \int_0^R dr$$

$$= 2\pi c R$$

$$\Rightarrow c = \frac{W}{2\pi R}$$

$$T_r = \mu P 2\pi r^2 dr$$

$$= \mu c \pi r^2 dr$$

$$= \mu c 2\pi r dr$$

$$T = \int_0^R T_r$$

$$= \int_0^R \mu c 2\pi r dr$$

$$\begin{aligned}
 T &= \int_0^R T_r \\
 &= \int_0^R \mu C 2\pi r dr \\
 &= \mu C \cdot 2\pi \int_0^R r dr \\
 &= \mu C 2\pi \frac{R^2}{2}
 \end{aligned}$$

$$= \mu \frac{W}{2\pi R} 2\pi \frac{R^2}{2}$$

$$T = \frac{1}{2} \mu W R$$

[Bf! 10/02/20]

Q// A vertical shaft 150mm diameter rotating at 100 rpm. Rest on a flat end. Foot step bearing carries a vertical load of 20kN assuming uniform pressure and 10-efficient of friction 0.05 estimate torque. Calculate the torque.

Given, $d = 150 \text{ mm}$
 $W = 20 \text{ kN}$ $\mu = 0.05$

$$W = 20 \text{ kN}$$

$$T = \frac{2}{3} \mu W R$$

$$= \frac{2}{3} \times 0.05 \times 20 \times 10^3 \times 1.75$$

$$= 50 \text{ kN.m}$$

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 100 \times 50}{60}$$

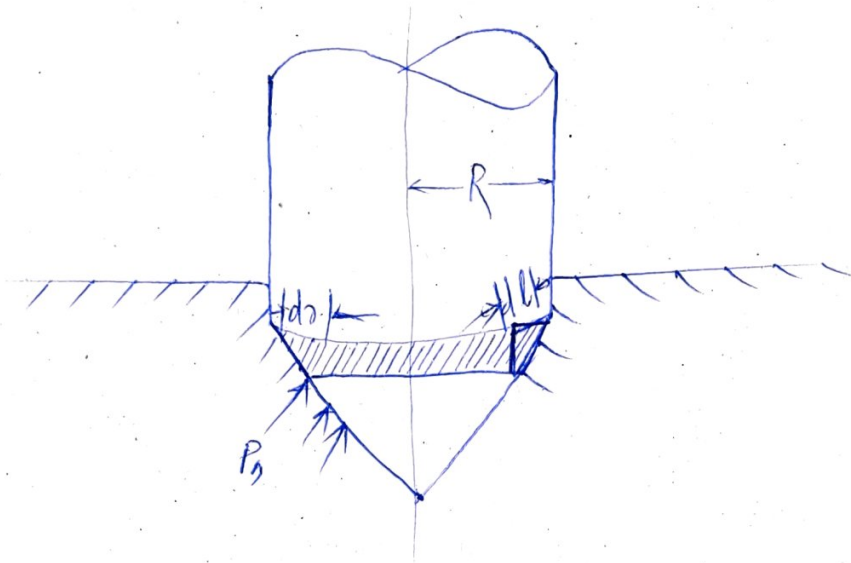
$$= 523.59 \text{ W}$$

$$P = \frac{2\pi NT}{60}$$

$$P = WT$$

$$W = \frac{2\pi N}{60}$$

Conical Pivot Bearing



Conical Pivot Bearing supporting a load W

P_n is the intensity of normal pressure

α is the semi angle of the cone

μ is the co-efficient of friction

R is the radius of the shaft

from geometry



$$\sin \alpha = \frac{dr}{dl}$$

$$\Rightarrow dl = \frac{dr}{\sin \alpha} = dr \csc \alpha$$

$$\begin{aligned} \text{Area of the ring} &= 2\pi r \times dl \\ &= 2\pi r \times dr \csc \alpha \end{aligned}$$

b) Considering Uniform Pressure

$$\delta W_n = P_n \times A$$

$$(\text{normal load}) = P_n \times 2\pi r dr \csc \alpha$$

Horizontal load

$$\delta W_h = \delta W_n \sin \alpha$$

$$= P_n \times 2\pi r dr \csc \alpha \times \sin \alpha$$

$$\boxed{\delta W = P_n 2\pi r dr}$$

$$W = \int_0^R \delta W = \int_0^R P_n 2\pi r dr$$

$$= P_n 2\pi \int_0^R r dr$$

$$= P_n 2\pi \left[\frac{R^2}{2} - \frac{0^2}{2} \right]$$

$$= P_n \times \pi \frac{R^2}{2}$$

$$= P_n \pi R^2$$

$$\boxed{W = \pi R^2 P_n}$$

$$F_r = \mu W_n$$

$$= \mu P_n 2\pi r dr \operatorname{cosec} \alpha$$

$$T_r = F_r \times r$$

$$= \mu P_n 2\pi r^2 dr \operatorname{cosec} \alpha$$

$$T = \int_0^R T_r$$

$$= \int_0^R \mu P_n 2\pi r^2 dr \operatorname{cosec} \alpha$$

$$= \mu P_n 2\pi \operatorname{cosec} \alpha \int_0^R r^2 dr$$

$$= \mu P_n 2\pi \operatorname{cosec} \alpha \left[\frac{r^3}{3} \right]_0^R$$

$$= \mu P_n 2\pi \operatorname{cosec} \alpha \left[\frac{R^3}{3} - \frac{0^3}{3} \right]$$

$$= \frac{2}{3} \pi \mu P_n \operatorname{cosec} \alpha R^3$$

$$= \cancel{2\pi} \frac{2\pi\mu}{3} \frac{W}{\pi R^2} \operatorname{cosec}\alpha R^3$$

$$= \frac{2}{3} \mu W R \operatorname{cosec}\alpha$$

Uniform Wear

We know that,

$$P_r = C/r$$

$$\delta W = P_r \times A$$

$$\Rightarrow \delta W = P_r \times 2\pi r dr$$

$$\Rightarrow \delta W = \frac{C}{r} \times 2\pi r dr$$

$$\delta W = 2\pi C dr$$

Total load,

$$W = \int_0^R 2\pi C dr = 2\pi C [r]_0^R$$

$$W = 2\pi C \cdot r$$

$$\Rightarrow \boxed{C = \frac{W}{2\pi r}}$$

Torque on the ring,

$$T_r = 2\pi \mu p r^2 d\theta$$

$$T_r = 2\pi \mu c \operatorname{cosec} \alpha r dr$$

$$T = \int_0^R 2\pi \mu c \operatorname{cosec} \alpha r dr$$

$$= 2\pi \mu c \operatorname{cosec} \alpha \int_0^R r dr$$

$$= 2\pi \mu c \operatorname{cosec} \alpha \left[\frac{r^2}{2} \right]_0^R$$

$$= 2\pi \mu c \operatorname{cosec} \alpha \left[\frac{R^2}{2} - \frac{0^2}{2} \right]$$

$$T = 2\pi \mu c \operatorname{cosec} \alpha \frac{R^2}{2}$$

Putting value of c

$$\Rightarrow T = \pi \mu \frac{W}{2\pi R} \operatorname{cosec} \alpha R^2 = \frac{1}{2} \mu W R \operatorname{cosec} \alpha$$
$$\Rightarrow T = \frac{1}{2} \mu W R \operatorname{cosec} \alpha$$

2// A conical pivot bearing support a vertical shaft of 200mm diameter if is subjected to a load of 30kN the angle of the cone is 120° . And co-efficient of friction is 0.025. Find the power lost in friction when the speed is 140 rpm. Assuming uniform pressure and uniform wear.

$$d = 200 \text{ mm} \quad R = 100 \text{ mm}$$

$$W = 30 \text{ kN}$$

$$2\alpha = 120^\circ$$

$$\alpha = 60^\circ \quad \omega = 2\pi \times \frac{140}{60}$$

$$\mu = 0.025 \quad = 14.66 \text{ rad/s}$$

$$T = \frac{2}{3} \mu W R \sec \alpha$$

$$= \frac{2}{3} \times 0.025 \times 30 \times 10^3 \times \frac{1}{\sin(60^\circ)}$$

$$= 577.35 \text{ N.m}$$

$$= 5$$

$$T = \frac{2}{3} \mu W R \operatorname{cosec} \alpha$$

$$= \frac{2}{3} \times 0.025 \times 100 \times 30 \times 10^3 \times \frac{1}{\sin(60^\circ)}$$

$$= 57.7 \text{ N.m}$$

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 100 \times 57.73}{60}$$

$$= 846.36 \text{ N}$$

Uniform wear

$$T = \frac{1}{2} \mu W R \operatorname{cosec} \alpha$$

$$= \frac{1}{2} \times 0.025 \times 30 \times 10^3 \times 100 \times \operatorname{cosec} 60^\circ$$

$$= 43.3 \text{ N.m}$$

$$P = T \cdot \omega$$

$$= 43.3 \times$$

$$14.66$$

$$= 634.79 \text{ W} = 634.8 \text{ W}$$

7

Considering a small layer having a radius r and thickness dr .

$$2\pi r \cdot dr$$

load on ring,

$$\begin{aligned}\delta W &= \text{Pressure} \times \text{area} \\ &= P \times 2\pi r dr\end{aligned}$$

Friction on ring

$$\begin{aligned}F_r &= \mu \cdot \delta W \\ &= \mu P \times 2\pi r dr \\ &= 2\pi \mu P r^2 dr\end{aligned}$$

Total Torque

$$T = \int_{r_2}^{r_1} 2\pi \mu p r^2 dr$$

$$= 2\pi \mu p \int_{r_2}^{r_1} r^2 dr$$

$$= 2\pi \mu p \left[\frac{r^3}{3} \right]_{r_2}^{r_1}$$

$$= 2\pi \mu p \left[\frac{r_1^3}{3} - \frac{r_2^3}{3} \right]$$

$$= 2\pi \mu p \left(\frac{r_1^3 - r_2^3}{3} \right)$$

$$= \frac{2}{3} \pi \mu p (r_1^3 - r_2^3)$$

we know

$$p = \frac{W}{\pi(r_1^2 - r_2^2)}$$

$$\Rightarrow T = \frac{2}{3} \cancel{\pi} \mu \frac{W}{\cancel{\pi(r_1^2 - r_2^2)}} \times (r_1^3 - r_2^3)$$

$$= \frac{2}{3} \cancel{\pi} \mu \left(W \times \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)$$

Total Torque

$$T = \int_{r_2}^{r_1} 2\pi \mu r \cdot r^2 dr$$

$$= 2\pi \mu r \int_{r_2}^{r_1} r^2 dr$$

$$= 2\pi \mu r \left[\frac{r^3}{3} \right]_{r_2}^{r_1}$$

$$= 2\pi\mu p \left(\frac{r_1^3 - r_2^3}{3} \right)$$

$$= \frac{2}{3} \pi\mu p (r_1^3 - r_2^3)$$

$$p = \frac{W}{\pi(r_1^2 - r_2^2)}$$

$$\Rightarrow T = \frac{2}{3} \pi\mu \frac{W}{\pi(r_1^2 - r_2^2)} (r_1^3 - r_2^3)$$

$$= \frac{2}{3} \mu W \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)}$$

Uniform Wear

Considering uniform wear

$$p \cdot r = C \text{ (constant)}$$

$$\Rightarrow p = \frac{C}{r}$$

$$\begin{aligned}\delta W &= \text{Pressure} \times \text{area} \\ &= \frac{c}{r} \times 2\pi r \cdot dr\end{aligned}$$

$$\begin{aligned}F_r &= \mu \cdot \delta W \\ &= \mu \frac{c}{r} \times 2\pi r dr\end{aligned}$$

$$\begin{aligned}T_r &= F_r \times r \\ &= \mu \frac{c}{r} \times r \times 2\pi r dr \\ &= \mu c \times 2\pi r dr\end{aligned}$$

Total Torque

$$\begin{aligned}T &= \int_{r_2}^{r_1} 2\pi \mu c r dr \\ &= 2\pi \mu c \int_{r_2}^{r_1} r dr \\ &= 2\pi \mu c \frac{r_1^2 - r_2^2}{2}\end{aligned}$$

$$= 2\pi \mu C \frac{r_1^2 - r_2^2}{2}$$

$$= \mu C C (r_1^2 - r_2^2)$$

Putting value of P

$$\Rightarrow T = \pi \mu \frac{W}{2\pi(r_1 - r_2)} (r_1^2 - r_2^2)$$

$$= \frac{1}{2} \pi \mu \frac{r_1^2 - r_2^2}{r_1 - r_2}$$

Q/A A single plate clutch with both sides effective is required to transmit 25 kW and 3000 rpm. Determine the outer and inner radius of frictional surface if the co-efficient of friction is 0.255. The ratio of radii is 1.25 and maximum

pressure not exceed 0.1 N/mm^2 . Also determine the axial thrust provided by the springs.

Given,

$$P = 25 \text{ kW} \quad \checkmark = 25 \times 10^3 \text{ W}$$

$$\eta = 2 \quad \checkmark$$

$$N = 3000 \text{ rpm.}$$

$$\mu = 0.255$$

$$\frac{r_1}{r_2} = 1.25$$

$$\Rightarrow r_1 = 1.25 r_2$$

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2 \times \pi \times 3000}{60} T$$

$$= 314.2 T \quad \checkmark$$

$$(1) P \cdot r_p = C$$

TO

$$\Rightarrow C = P \times 1.25 r_2$$

$$W = 2\pi C \cdot dr$$

$$= 2\pi \times P \times 1.25 r_2 (1.25 r_2 - r_2)$$

$$R = \frac{r_1 + r_2}{2} = \frac{1.25 r_2 + r_2}{2}$$

$$= \frac{2.25 r_2}{2}$$

$$R = 1.125 r_2 \quad \checkmark$$

$$\frac{250}{2}$$

$$T = \eta \mu WR$$

$$= 2 \times 0.25 \times 7.85 r_2 (r_1 - r_2) \times 1.25 r_2$$

$$W = 2\pi C$$

$$W = 2\pi C (r_1 - r_2)$$

$$= 2\pi \times 0.125 r_2 (1.25 r_2 - r_2)$$

$$= \cancel{(2\pi \times 0.125 r_2 \times 1.25 r_2)} - (2\pi \times 0.125 r_2 \times r_2)$$

$$= 2\pi \times 0.125 r_2 (0.25 r_2)$$

$$= 0.196 r_2^2$$

$$R = \frac{1.25 r_2 + r_2}{2}$$

$$= \cancel{2.25} \times 1.125 r_2$$

$$T = \mu WR \times n$$

$$\Rightarrow 79.6 \times 10^3 = 0.255 \times 0.196 (r_2)^2 \times 1.125 r_2 \times 2$$

$$\Rightarrow 79.6 \times 10^3 \times 0.255 \times 0.2205 r_2^3 \times 2$$

$$\Rightarrow \frac{79.6 \times 10^3}{0.255 \times 2} = 0.2205 r_2^3$$

$$\Rightarrow r_2^3 = \sqrt[3]{\frac{79.6 \times 10^3}{0.225 \times 2 \times 0.2205}}$$

$$\Rightarrow r_2 = 92.91 \text{ mm}$$

$$r_1 = 1.25 r_2$$

$$= 1.25 \times 92.91 = 116.13 \text{ mm}$$

$$W = 2\pi C (r_1 - r_2)$$

$$= 2 \times \pi \times 0.125 \times 92.91 \times (116.13 - 92.91)$$

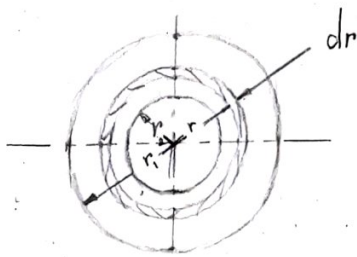
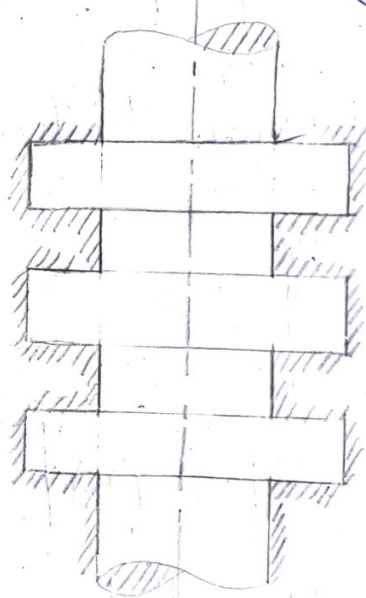
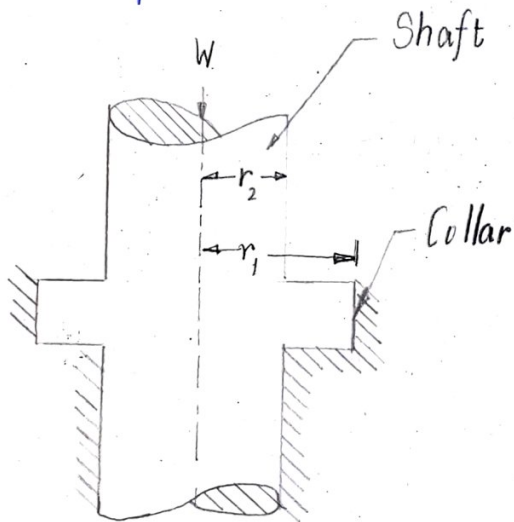
$$= 1694.39 \text{ N}$$

$$W = 2\pi \times 0.125 \times 0.25 \times r_2^2$$

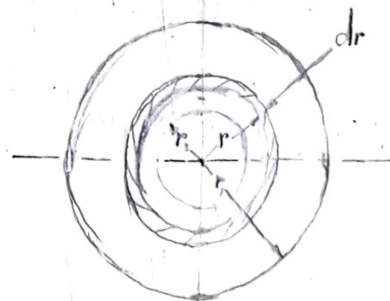
$$= 1694.94 \text{ N}$$

Flat Collar bearing

- Collar bearings are used to take the axial thrust of the rotating shafts.
- They may be a single collar or multiple collar bearings.
- The collar is also known as thrust bearings.



Single collar bearing



Multiple collar bearing

Let's

Consider a single flat collar bearing supporting a shaft.

Let

r_1 = External radius of the collar

r_2 = Internal radius of the collar

∴ Area of the bearing surface,

$$A = \pi [(r_1)^2 - (r_2)^2]$$

Considering Uniform Pressure

$$p = \frac{W}{A}$$

$$p = \frac{W}{\pi [(r_1)^2 - (r_2)^2]}$$

————— (1)

We know that, frictional torque

$$T_r = 2\pi \mu \cdot p \cdot r^2 \cdot dr$$

Total frictional torque

$$T = \int_{r_2}^{r_1} 2\pi \mu \cdot p \cdot r^2 \cdot dr$$

$$= 2\pi \mu \cdot p \left[\frac{r^3}{3} \right]_{r_2}^{r_1}$$

$$= 2\pi \mu \cdot p \left[\frac{(r_1)^3 - (r_2)^3}{3} \right]$$

substituting the value of p from eqⁿ (1)

$$T = 2\pi \mu \frac{W}{\pi [(r_1)^2 - (r_2)^2]} \left[\frac{(r_1)^3 - (r_2)^3}{3} \right]$$

$$T = \frac{2}{3} \times \mu \cdot W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

In case of a multi-collared bearing

$$p = \frac{\text{Load}}{\text{No. of collars} \times \text{Bearing area of one collar}}$$

$$= \frac{W}{n \pi [(r_1)^2 - (r_2)^2]}$$

Total torque transmitted

$$T = \frac{2}{3} \mu W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

Considering uniform wear
load transmitted
 $\delta W = p_s \cdot 2\pi r \cdot dr$

$$= \frac{C}{s} \times 2\pi r dr$$

$$= 2\pi C \cdot dr$$

Total load transmitted,

$$W = \int_{r_2}^{r_1} 2\pi C \cdot dr$$

$$= 2\pi C [r]_{r_2}^{r_1}$$

$$= 2\pi C (r_1 - r_2)$$

$$\Rightarrow C = \frac{W}{2\pi (r_1 - r_2)} \quad \text{--- (ii)}$$

Frictional torque on the ring,

$$T_s = \mu \delta W \cdot r$$

$$= \mu \times 2\pi C dr \cdot r$$

$$= 2\pi \mu \cdot C \cdot r dr$$

Total frictional load on the bearing

$$T = \int_{r_2}^{r_1} 2\pi \mu C \cdot r \cdot d\sigma$$

$$= 2\pi \mu \cdot C \left[\frac{r^2}{2} \right]_{r_2}^{r_1}$$

$$= 2\pi \mu C \left[\frac{(r_1)^2 - (r_2)^2}{2} \right]$$

$$= \pi \mu \cdot C [(r_1)^2 - (r_2)^2]$$

Substituting value from equation (ii)

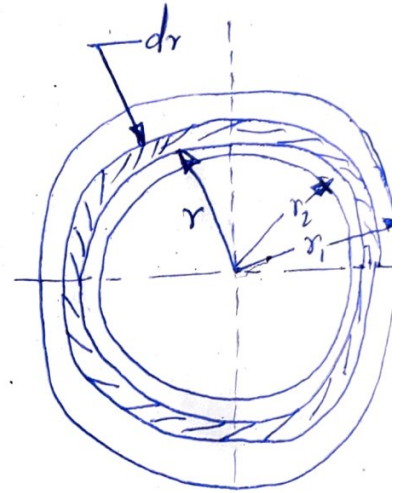
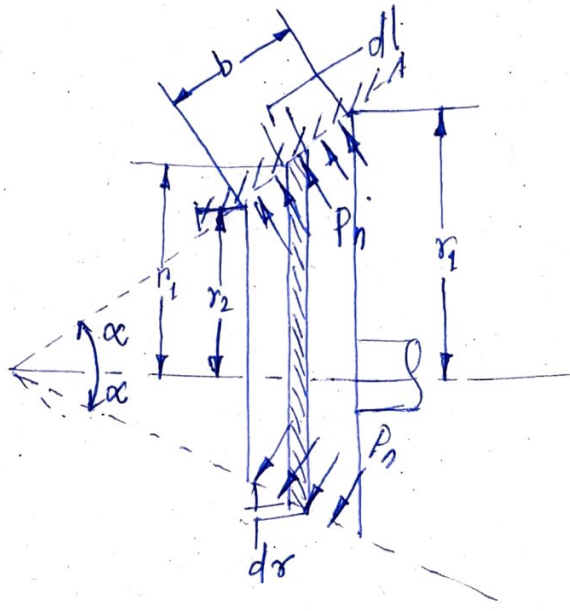
$$T = \pi \mu \times \frac{W}{2\pi(r_1 - r_2)} [(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu \cdot W (r_1 + r_2)$$

Let

Cone Clutch

$P_n = \text{Intensi.}$



$P_n = \text{normal pressure}$

$r_1 = \text{Outer radius}$

$r_2 = \text{Inner radius}$

$$R = \frac{r_1 + r_2}{2}$$

$\alpha = \text{semi angle}$

$\mu = \text{co-efficient of friction}$

$b = \text{Width of the contact surface}$

$dr = \text{thickness of the ring}$

$dl = \text{length of the ring}$

then,

$$dl = dr \operatorname{cosec} \alpha$$

Area of the ring

$$A = 2\pi r dl \\ = 2\pi r dr \operatorname{cosec} \alpha$$

Considering Uniform Pressure

$$\delta W_n = \text{Normal pressure} \times \text{Area of the ring}$$

$$= P_n \times 2\pi r \cdot dr \operatorname{cosec} \alpha$$

axial $\equiv$ load acting on the ring

$$\delta W = \text{Horizontal component of } \delta W_n$$

$$= \delta W_n \times \sin \alpha$$

$$= P_n \times 2\pi r \cdot dr \operatorname{cosec} \alpha \times \sin \alpha$$

$$= 2\pi \times P_n r \cdot dr$$

Total axial load

$$W = \int_{r_2}^{r_1} 2\pi P_n r \cdot dr = 2\pi P_n \left[\frac{r^2}{2} \right]_{r_2}^{r_1}$$

$$= 2\pi p_n \left[\frac{(r_1)^2 - (r_2)^2}{2} \right]$$

$$= \pi p_n [(r_1)^2 - (r_2)^2]$$

$$p_n = \frac{W}{\pi [(r_1)^2 - (r_2)^2]}$$

————— (1)

We know that,

$$F_r = \mu \delta W_n$$

$$= \mu p_n \times 2\pi r dr \operatorname{cosec} \alpha$$

Frictional Torque

$$T_r = F_r \times r$$

$$= \mu p_n \times 2\pi r dr \operatorname{cosec} \alpha \times r$$

$$= \mu p_n \times 2\pi r^2 dr \operatorname{cosec} \alpha$$

Total frictional torque

$$T = \int_{r_2}^{r_1} 2\pi \mu p_n \operatorname{cosec} \alpha r^2 dr$$

$$= 2\pi \mu p_n \operatorname{cosec} \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3} \right]$$

Substituting the value of P_n from equation (1)

$$T = 2\pi \mu \times \frac{W}{\pi [(r_1)^2 - (r_2)^2]} \times \csc \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3} \right]$$

$$= \frac{2}{3} \times \mu W \csc \alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

Ex. A cone clutch with ~~asbestos~~ asbestos friction lining transmits 25 kW at 600 rpm. The coefficient is 0.25 and the maximum intensity of pressure is 0.25 N/mm². The semi cone angle is 12.5°. The outer diameter of friction lining is 250 mm. Considering uniform wear theory. Determine

- i. the inner diameter of the friction lining.
- ii. the face width of friction lining and
- iii. the force required to engage to the clutch.

Given,

$$P = 25 \text{ kW} = 25 \times 10^3 \text{ N}$$

$$N = 600 \text{ rpm}$$

$$\mu = 0.25$$

$$P_{\max} = 0.25 \text{ N/mm}^2 =$$

$$\alpha = 12.50$$

$$2r_1 = D_1 = 250 \text{ mm}$$

Torque transmitted, by the clutch

We know that,

$$T = \frac{P}{\omega}$$

$$= \frac{25 \times 10^3}{2\pi \times \frac{600}{60}}$$

$$= 397.88 \text{ N-m}$$

$$= 397.9 \text{ N-m}$$

— (i)

$$T_f = \frac{2\pi\mu}{\sin\alpha} P_{\max} * r^2 \left(\frac{r_1^2 - r_2^2}{2} \right)$$

$$= \frac{2\pi \times 0.25}{\sin(12.50)} 0.25 \times r_2^2 \left(\frac{r_1^2 - r_2^2}{2} \right) \leftarrow (ii)$$

$$= 0.9072 \times r_2 (125^2 - r_2^2)$$

Equating the relation i and ii

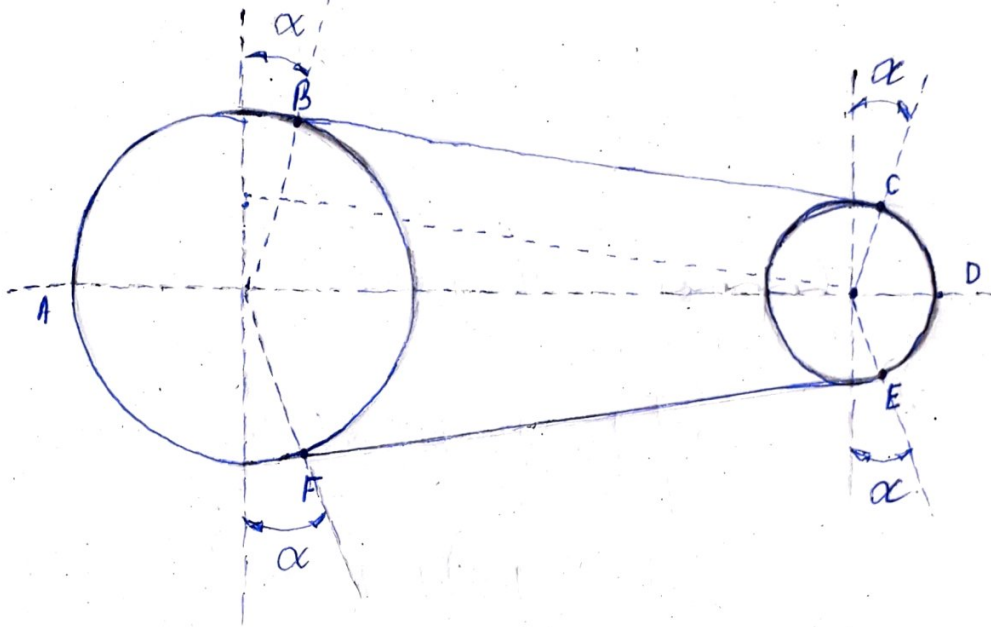
$$r_2 (125^2 - r_2^2) = 4038602.3$$

$$\Rightarrow \cancel{125^2 \times r_2} = r_2 (125 + r_2)(125 - r_2) = 4038602.3$$

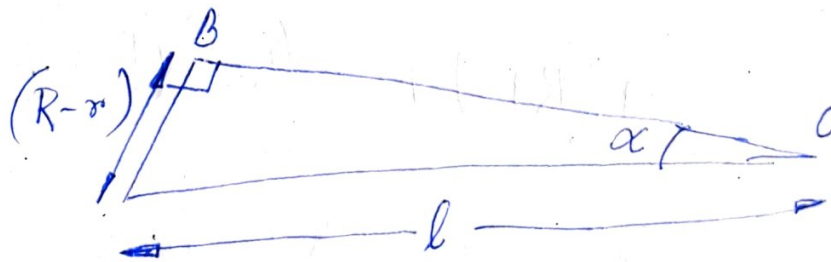
$$\Rightarrow \cancel{r_2} \cancel{125} (1 \Rightarrow r_2 (15625 - r_2^2) = 4038602.3$$

$$\Rightarrow 15625 r_2 - r_2^3 = 4038602.3$$

Open Belt Drive



$$\begin{aligned} \text{Length of open belt drive} &= 2 \left[\text{Arc AB} + BC + \text{Arc CB} \right] \\ &= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + BC + \left(\frac{\pi}{2} - \alpha \right) r \right] \end{aligned}$$



$$\begin{aligned} BC &= \sqrt{l^2 - (R-r)^2} \\ &= l \sqrt{1 - \frac{(R-r)^2}{l^2}} \\ &= l \left[1 - \frac{(R-r)^2}{l^2} \right]^{\frac{1}{2}} \end{aligned}$$

Using Binomial distribution

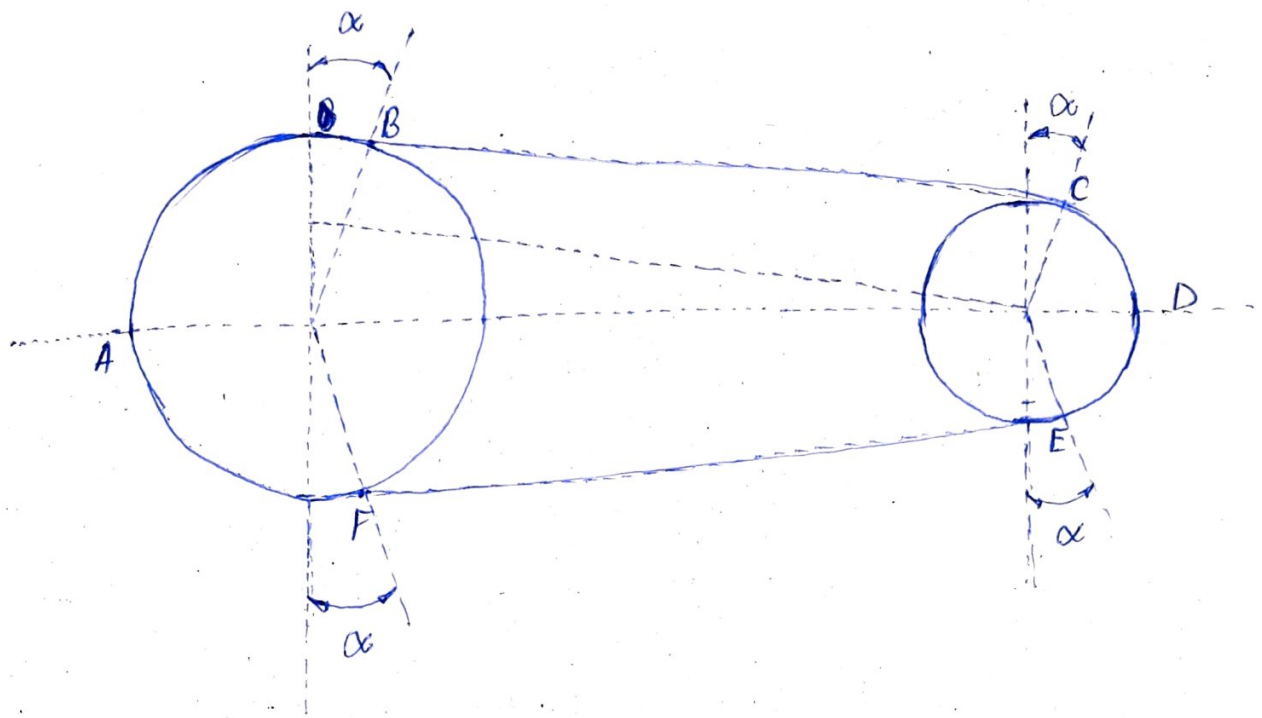
$$BC = l \left[1 - \frac{(R-r)^2}{2l^2} \right] \quad \sin \alpha = \frac{(R-r)}{l}$$

$\alpha = \frac{R-r}{l}$

Length of open belt drive

$$\begin{aligned}
 &= 2 \left[\text{Arc AB} + BC + \text{Arc CB} \right] \\
 &= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + l - \frac{(R-r)^2}{2l} + \left(\frac{\pi}{2} - \alpha \right) r \right] \\
 &= (\pi + \alpha) l + 2l - \frac{(R-l)^2}{l} + (\pi - 2\alpha) r \\
 &= \pi(R+l) + 2\alpha(R-l) - \frac{(R-l)^2}{l} + 2l \\
 &= \pi(R+r) + \frac{2(R-l)(R-\alpha)}{l} - \frac{(R-l)^2}{l} + 2l \\
 &= \pi(R+r) + \frac{(R-r)^2}{l} + 2l
 \end{aligned}$$

$$= \pi(R+r) + \frac{(R-r)^2}{l} + 2l$$



Length of open belt drive

$$= 2 [\text{Arc AB} + \text{BC} + \text{Arc CB}]$$

$$= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + \text{BC} + \left(\frac{\pi}{2} - \alpha \right) r \right] \quad \text{--- (1)}$$

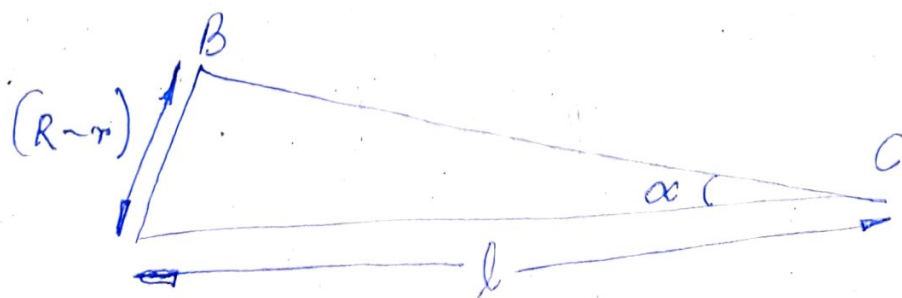
From the geometry of the figure,

$$\sin \alpha = \frac{R-r}{l}$$

Since α is very small,

$$\sin \alpha = \alpha = \frac{R-r}{l}$$

Here,



$$\begin{aligned}
 BC &= \sqrt{l^2 - (R-r)^2} \\
 &= l \sqrt{1 - \frac{(R-r)^2}{l^2}} \\
 &= l \left(1 - \frac{(R-r)^2}{l^2} \right)^{1/2}
 \end{aligned}$$

Using Binomial distribution,

$$BC = l \left[1 - \frac{(R-r)^2}{2l^2} \right]$$

~~From eqⁿ~~ 

We know that,

length of the open belt drive

$$= 2 \left[\text{Arc AB} + BC + \text{Arc CB} \right]$$

$$= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + BC + \left(\frac{\pi}{2} - \alpha \right) r \right]$$

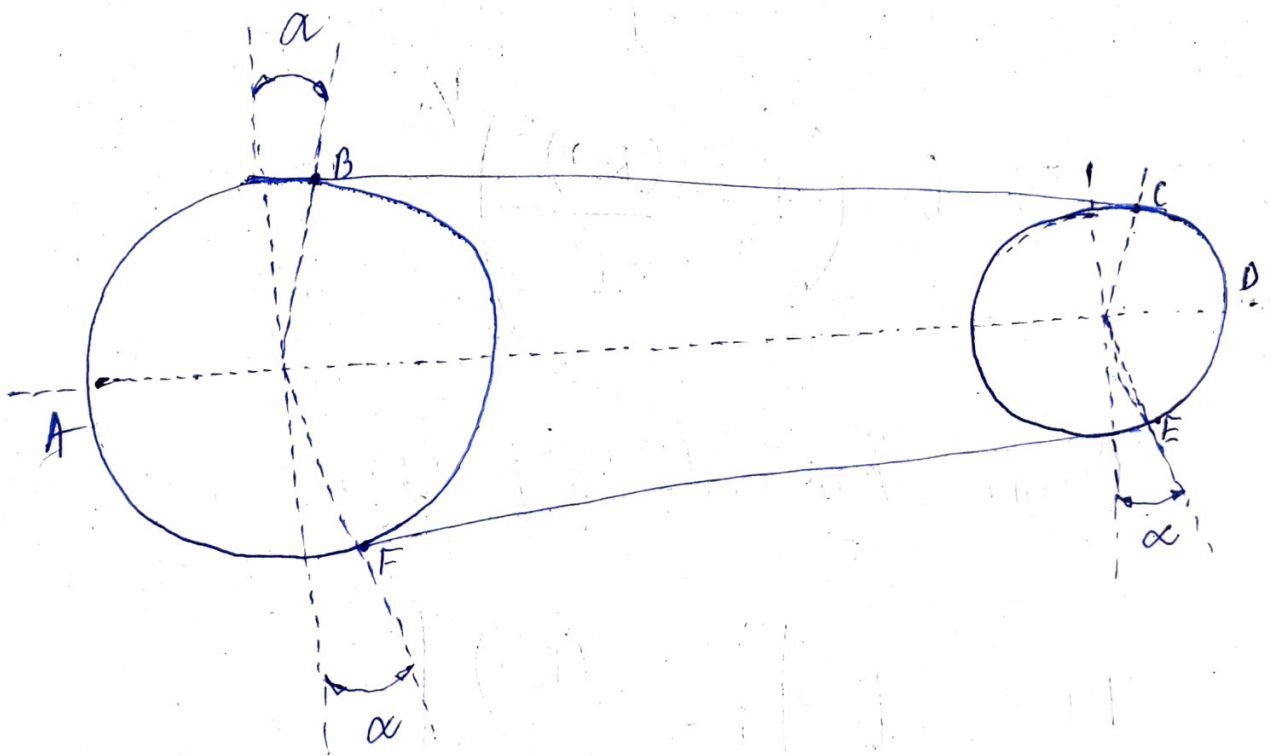
$$= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + l - \frac{(R-r)^2}{2l} + \left(\frac{\pi}{2} - \alpha \right) r \right]$$

$$= (\pi + \alpha)R + 2l - \frac{(R-r)^2}{l} + (\pi - 2\alpha)r$$

$$= \pi(R+r) + 2\alpha(R-r) - \frac{(R-r)^2}{l} + 2l$$

$$= \pi(R+r) + 2\frac{(R-r)(R-r)}{l} - \frac{(R-r)^2}{2} + 2l$$

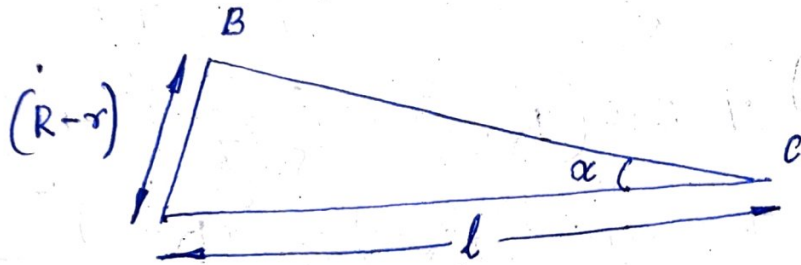
$$L = \pi(R+r) + \frac{(R-r)^2}{l} + 2l$$



Length of open belt drive

$$= 2 [\text{Arc AB} + \text{BC} + \text{Arc CB}]$$

$$= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + \text{BC} + \left(\frac{\pi}{2} - \alpha \right) r \right]$$



$$\begin{aligned}
 BC &= \sqrt{l^2 - (R-r)^2} \\
 &= l \sqrt{1 - \frac{(R-r)^2}{l^2}} \\
 &= l \left(1 - \frac{(R-r)^2}{l^2} \right)^{1/2}
 \end{aligned}$$

Using Binomial distribution

$$BC = l \left[1 - \frac{(R-r)^2}{2l^2} \right]$$

from the geometry of figure,

$$\sin \alpha = \frac{R-r}{l}$$

$\sin \alpha$ is very small

$$\sin \alpha = \alpha = \frac{R-r}{l}$$

We know that,

length of the open belt drive,

$$= 2 [\text{Arc AB} + BC + \text{Arc CB}]$$

$$= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + BC + \left(\frac{\pi}{2} - \alpha \right) r \right]$$

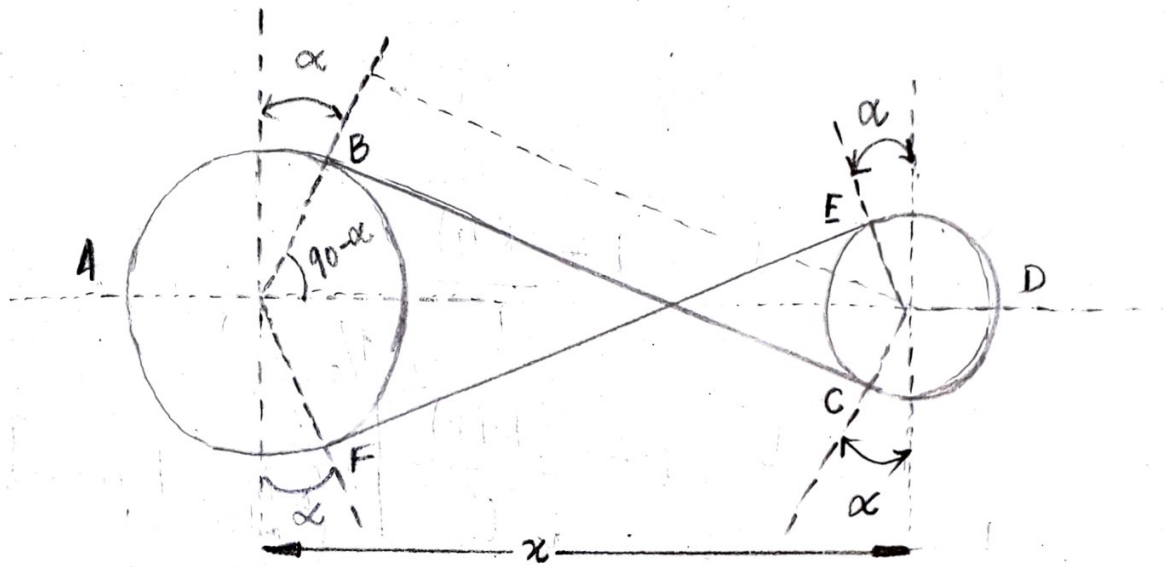
$$= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + l - \frac{(R-r)^2}{2l} + \left(\frac{\pi}{2} - \alpha \right) r \right]$$

$$= (\pi + \alpha) R + 2l - \frac{(R-r)^2}{l} + (\pi - 2\alpha) r$$

$$= \pi(R+r) + 2\alpha(R-r) - \frac{(R-r)^2}{l} + 2l$$

$$= \pi(R+r) + 2(R-r)(R-r) - \frac{(R-r)^2}{2} + 2l$$

$$\boxed{L = \pi(R+r) + \frac{(R-r)^2}{l} + 2l}$$

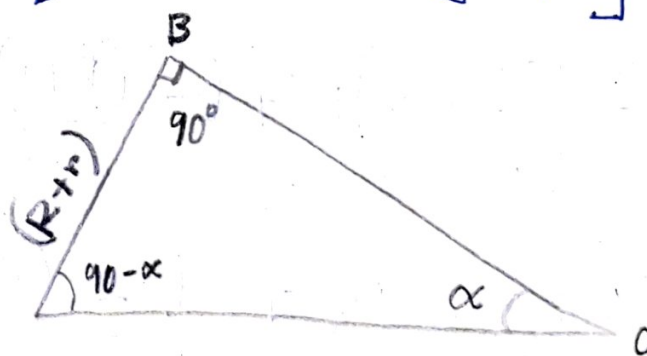


Length of cross-belt drive

$$= 2 \left[\text{Arc AB} + BC + \text{Arc CD} \right]$$

$$= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + BC + \left(\frac{\pi}{2} + \alpha \right) r \right] \quad \text{--- (1)}$$

from the geometry



$$\sin \alpha = \frac{(R+r)}{L}$$

Hence,

$$\sin \alpha = \alpha = \frac{(R+r)}{L}$$

$$\begin{aligned}
 BC &= \sqrt{(R)^2 - (R+l)^2} \\
 &= l \sqrt{1 - \frac{(R+r)^2}{l^2}} \\
 &= l \left[1 - \frac{(R+r)^2}{l^2} \right]^{1/2} \quad \left(\text{using binomial distribution} \right)
 \end{aligned}$$

$$BC = \left[l - \frac{(R+r)^2}{l} \right]$$

From -- (1),

$$= 2 \left[\left(\frac{\pi}{2} + \alpha \right) R + l - \frac{(R+r)^2}{2l} + \left(\frac{\pi}{2} + \alpha \right) r \right]$$

$$= \pi R + 2\alpha R + 2l - \frac{(R+r)^2}{2l}$$

$$+ \pi r + 2\alpha r$$

$$= \pi(R+l) + 2l(R+r) + 2l - \frac{(R+l)^2}{2l}$$

$$= \pi(R+r) + 2 \frac{(R+r)^2}{2} - (R+r) + 2l - \frac{(R+r)^2}{2l}$$

$$= \pi(R+r) + 2 \frac{(R+r)^2}{2} + 2l - \frac{(R+r)^2}{2l}$$

$$= \pi(R+r) + \frac{(R+r)^2}{l} + 2l$$

⇒ Length of the cross-belt drive

$$L = \pi(R+r) + \frac{(R+r)^2}{l} + 2l$$

'R' and 'r' is the Radii of the larger and smaller pulleys.

'x' is the distance between the two centres of two pulleys

'L' is the total length of the belt.

- Belt transmission is ~~complet~~ comparatively simple, trouble free, easy to be served, maintained and have relatively low initial cost.
- Belt can connect non-parallel shafts also.
- Belts ~~can~~ do not have space constraints, lubrication or heating issues.
- Since belts are flexible material there is inbuilt shock protection.
- Mainly belt drive are flat belt drive and V-belt drive.
- Flat belts although simple and economical they have relatively less grip and make more noise.

Questions and Answers

Q.1

Why belts are called 'Flexible Drives'?
What advantages and limitations they offer over the gears?

→ In belt drives there is no direct contact between the driving and driven members, but the connection is through a flexible link called Belt, hence this device is named as Flexible drive. The belt can absorb the shock and also permits the misalignment due to its flexibility.

Following are the advantages it offers over the gear drive.

→ Belt drives can transmit power over a long distance which is not

feasible with gears.

Since the intermediate link is long and flexible. It can absorb the shocks and vibrations

Limitations:-

- There are chances of slip in belts, hence belt drives are not positive drives and not 100% reliable.
- They cause more noise than gears.
- Large velocity can not be obtained.

Q.2

State the advantages and disadvantages of flat over V-belt?

Advantages of flat belt over V-belt

- Flat belt are relatively cheap and simple in construction
- Flat belts can be used for long central distances
- Different velocity ratio can be ~~used for long central distances.~~ obtained with stepped pulleys.
- Easy to replace or adjust length

Disadvantages of flat belt over V-belt

- The power transmission capacity is low

- less frictional grip with results in changes of slippage.
- Make more noise.
- Can not be used for very high velocity ratio.
- Need more space due to width of belt as compared to v for same power transmission

Problems on belt drives

Example 11.1

An engine, running at 150 rpm drives a line shaft by means of a belt.

The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamic

shaft. Find the speed of the dynamic shaft, when 1. there is no slip 2. there is a slip of 2% at each drive.

Given, $N_1 = 150 \text{ rpm}$.

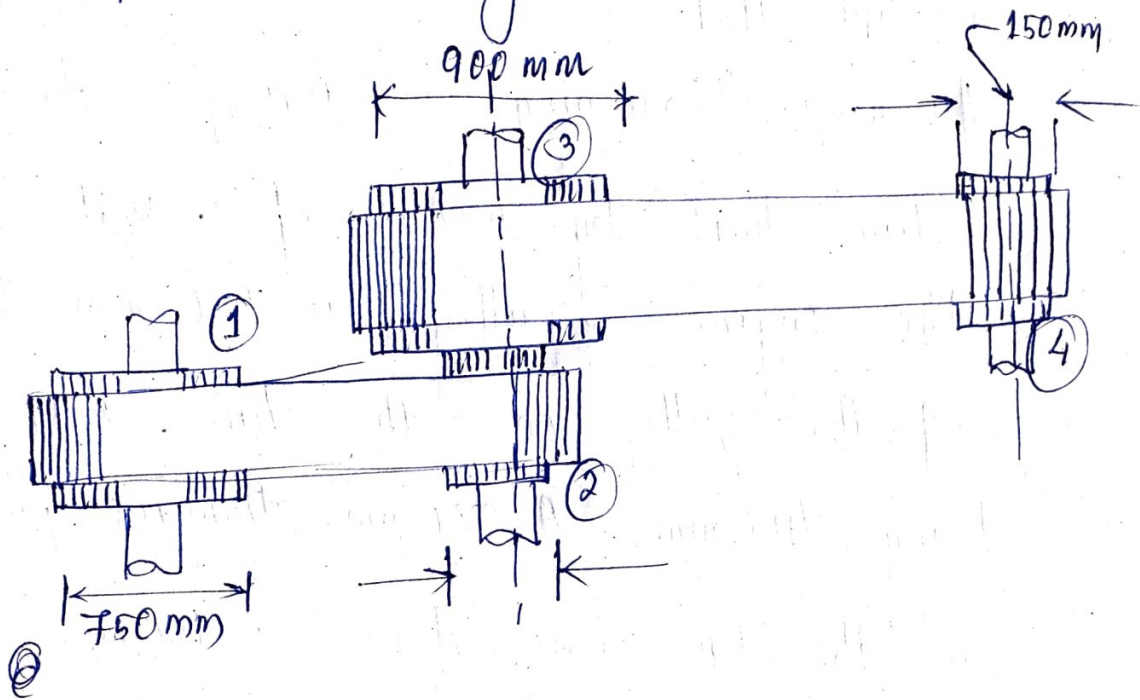
$$d_1 = 750 \text{ mm}$$

$$d_2 = 450 \text{ mm}$$

$$d_3 = 900 \text{ mm}$$

$$d_g = 500 \text{ mm}$$

N_g = speed of dynamo



When there is no slip

We know that

$$\frac{N_1}{N_2} = \frac{d_2 \times d_4}{d_1 \times d_3}$$

$$\Rightarrow \frac{N_g}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

$$\Rightarrow N_g = 150 \times 10 = 1500 \text{ r.p.m.}$$

Where there is a slip of 2% at each drive.

We know that,

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \left(1 - \frac{s_1}{100}\right) \left(1 - \frac{s_2}{100}\right)$$

$$\Rightarrow \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} \left(1 - \frac{2}{100}\right) \left(1 - \frac{2}{100}\right)$$
$$= 9.6$$

$$N_4 = 150 \times 9.6$$
$$= 1440 \text{ rpm}$$

Example 11.2

The power is transmitted from a pulley 1m diameter running at 200 rpm to a pulley 2.25 diameter by means of a belt. Find the speed lost by the driven pulley as a result of creep, if the stress on

tight and slack side of the belt is 1.4 MPa and 0.5 MPa respectively. The Young's modulus for the material of the belt is 100 MPa.

Solution,

Given,

$$d_1 = 1 \text{ m.}$$

$$N_1 = 200 \text{ rpm.}$$

$$d_2 = 2.25 \text{ m}$$

$$\sigma_1 = 1.4 \text{ MPa}$$

$$= 1.4 \times 10^6 \text{ N/m}^2$$

$$\sigma_2 = 0.5 \text{ MPa}$$

$$= 0.5 \times 10^6 \text{ N/m}^2$$

$$E = 100 \text{ MPa}$$

$$= 100 \times 10^6 \text{ N/m}^2$$

Let N_2 = Speed of the driven pulley

Neglecting creep, we know that

$$\frac{N_2}{N_1} = \frac{d_2}{d_1}$$

$$\begin{aligned}\Rightarrow N_2 &= N_1 \times \frac{d_1}{d_2} \\ &= 200 \times \frac{1}{2.25} \\ &= 88.9 \text{ rpm}\end{aligned}$$

Considering creep.

We know that,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

$$\begin{aligned}\Rightarrow N_2 &= 200 \times \frac{1}{2.25} \times \frac{100 \times 10^6 + \sqrt{0.5 \times 10^6}}{100 \times 10^6 + \sqrt{1.4 \times 10^6}} \\ &= 88.7 \text{ r.p.m.}\end{aligned}$$

$\therefore$ Speed lost by driven pulley due to creep.

$$= 88.9 - 88.7 = 0.2 \text{ rpm}$$

Q. 11.3

A shaft which rotates at a constant speed of 160 rpm. is connected by belting to a parallel shaft 720 mm apart, which has to run at 60, 80 and 100 rpm. The smallest pulley on the driven shaft is 40 mm in radius. Determine the remaining radii of the two stepped pulleys for

1. a crossed belt.

2. an open belt. Neglect belt thickness and slip.

Given,

$$N_1 = N_3 = N_5 = 160 \text{ rpm}$$

$$x = 720 \text{ mm}$$

$$N_2 = 60 \text{ rpm}$$

$$N_4 = 80 \text{ r.p.m}$$

$$N_6 = 100 \text{ r.p.m}$$

$$r_1 = 40 \text{ mm}$$

Let r_2, r_3, r_4 and r_6 be the radii of the pulleys 2, 3, 4, 5 and 6 respectively.

We know that for pulley 1 and 2

$$\frac{N_2}{N_1} = \frac{r_1}{r_2}$$

$$r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm}$$

and for pulleys 3 and 4.

$$\frac{N_4}{N_3} = \frac{r_3}{r_4}$$

$$\begin{aligned} \Rightarrow r_4 &= r_3 \times \frac{N_3}{N_4} \\ &= r_3 \times \frac{160}{80} \\ &= 2r_3 \end{aligned}$$

We know for a cross-belt drive

$$r_1 + r_2 = r_3 + r_4 = r_5 + r_6 = 40 + 106.7 \\ = 146.7 \text{ mm}$$

$$r_2 + 2r_3 = 146.7$$

$$r_3 = 146.7/3 = 48.9 \text{ mm}$$

$$r_4 = 2r_3 = 2 \times 48.9 = 97.8 \text{ mm}$$

Now for pulleys 5 and 6

$$\frac{N_6}{N_5} = \frac{r_5}{r_6}$$

$$\Rightarrow r_6 = r_5 \times \frac{N_5}{N_6} \\ = r_5 \times \frac{160}{100} \\ = 1.6 r_5$$

From equation (i),

$$r_5 + 1.6 r_5 = 146.7$$

$$\Rightarrow r_5 = 146.7 / 2.6 = 56.4 \text{ mm}$$

$$r_6 = 1.6 r_3 = 1.6 \times 56.4 = 90.2 \text{ mm}$$

For an open belt

We know that for pulley 1 and 2,

$$\frac{N_2}{N_1} = \frac{r_1}{r_2} \Rightarrow r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm}$$

and for pulleys 3 and 4.

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \Rightarrow r_4 = r_3 \times \frac{N_3}{N_4} = r_3 \times \frac{160}{80} = 2r_3$$

We know that length of belt for an open belt drive,

$$\begin{aligned} L &= \pi(r_1 + r_2) + \frac{(r_2 - r_1)^2}{x} + 2x \\ &= \pi(40 + 106.7) + \frac{(106.7 - 40)^2}{720} + 2 \times 720 \\ &= 1907 \text{ mm} \end{aligned}$$

Since the length of the belt in an open belt drive is constant, therefore for pulley 3 and 4 length of the belt.

$$\begin{aligned}
 1907 &= \pi (r_3 + r_4) + \frac{(r_4 - r_3)^2}{\pi} + 2x \\
 &= \pi (r_3 + 2r_3) + \frac{(2r_3 - r_3)^2}{720} + 2 \times 720 \\
 &= 9.426 r_3 + 0.0014 (r_3)^2 + 1440
 \end{aligned}$$

$$\Rightarrow 0.0014 (r_3)^2 + 9.426 r_3 - 467 = 0$$

$$r_3 = \frac{-9.426 \pm \sqrt{(9.426)^2 + 4 \times 0.0014 \times 467}}{2 \times 0.0014}$$

$$= \frac{-9.426 \pm 9.564}{0.0028}$$

$$= 49.3 \text{ mm}$$

$$r_4 = 2r_3 = 2 \times 49.3 = 98.6 \text{ mm}$$

Now for pulley a. 5 and 6

$$\begin{aligned}
 \frac{N_6}{N_5} &= \frac{r_5}{r_6} \Rightarrow r_6 = \frac{N_5}{N_6} \times r_5 = \frac{160}{100} \times r_5 \\
 &= 1.6 r_5
 \end{aligned}$$

length of the belt (L)

$$1907 = \pi (r_5 + r_6) + \frac{(r_6 - r_5)^2}{\pi} + 2x$$

$$= \pi (r_5 + 1.6r_5) + \frac{(1.6r_5 - r_5)^2}{720} + 2 \times 720$$

$$= 8.17r_5 + 0.0005 (r_5)^2 + 1440$$

$$\Rightarrow 0.0005 (r_5)^2 + 8.17r_5 - 467 = 0$$

$$\therefore r_5 = \frac{-8.17 \pm \sqrt{(8.17)^2 + 4 \times 0.0005 \times 467}}{2 \times 0.0005}$$

$$= \frac{-8.17 \pm 8.23}{0.001} = 60 \text{ mm.}$$

$$r_6 = 1.6r_5 = 1.6 \times 60 = 96 \text{ mm (Ans)}$$

Example 11.4

Find the power transmitted by belt running over a pulley of 600mm diameter at 200 rpm. Coefficient of friction between the belt and the pulley is 0.25 angle of loop 160° and maximum tension in the belt is 2500 N.

Given,

$$d = 600 \text{ mm} \\ = 0.6 \text{ m}$$

$$N = 200 \text{ rpm}$$

$$\mu = 0.25$$

$$\theta = 160^\circ = 160 \times \pi \times 180 = 2.793 \text{ rad}$$

$$T_1 = 2500 \text{ N}$$

We know that velocity of the belt,

$$v = \frac{\pi d \cdot N}{60} = \frac{\pi \times 0.6 \times 200}{60} \\ = 6.284 \text{ m/s}$$

T_2 = Tension in the slack side of the belt.

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu \cdot \theta \\ = 0.25 \times 2.793 \\ = 0.6982$$

$$\log \left(\frac{T_1}{T_2} \right) = \frac{0.6982}{2.3} = 0.3036$$

$$\frac{T_1}{T_2} = 2.01$$

$$T_2 = \frac{T_1}{2.01} = \frac{2500}{2.01} = 1244 \text{ N}$$

We know that the power transmitted by the belt,

$$P = (T_1 - T_2) v$$

$$= (2500 - 1244) \times 6.284$$

$$= 7890 \text{ W}$$

$$= 7.89 \text{ kW}$$

About Belt Drive

Q// Why belts are called 'Flexible Drives' What advantages and limitations they offer over gears?

Ans :- In belt drives there is no direct contact between the driving and ~~there is no~~ driven members, but the connection is through a flexible link called Belt, Hence this drive is name as flexible drive.

The belt can absorb the shocks and also permits the misalignment due to its flexibility.

Advantages:

- It offers over the gear drive. ~~is~~
~~name as Flexible drive. The belt can~~
~~absorb the shock~~
 - Belt drives can transmit power over a long distance which is not feasible with gears.
 - Since the intermediate link is long and flexible it can absorb the shocks and vibrations.
 - Due to flexibility there is no space and tolerance ~~con~~ constraints like gears.
- Belt drives are very cheaper as compared to the gear drives
- Belt drives are simple to design.

Limitations :

- There are chances of slip in belts, hence belt drives are not positive drives and not 100% reliable.
- They cause more noise than gears
- Belt drives occupy more space as compared to gear trains.

Example 11.1.

64/

ABOUT BELT DRIVE

- The rotation motion is the ideal and simplest means of transmission of mechanical power with negligible losses.
- Rotational motion can be transmitted from one mechanical element to the other with the help of certain system known as transmission systems or drives.
- These systems are driven by a prime mover or transmit the rotational motion to the various parts of a machine within itself. Usually, shafts are employed to transmit rotational motion.
- Belt Drives are a type of frictional drives used for transmitting powers from one shaft to another by means of pulleys which rotates

at the same speed or at the different speed.

→ The portion of the belt which is having less tension is called tight side.

→ The effective pulling power of the belt that causes the rotation of the driven pulley is the difference in tension on the slack and tight side.

→ The tensions in the tight and slack sides of the belt drives have to be arranged such that the slack side comes above and the tight side comes below the pulleys.

- This arrangement increases the angled contact of the belt on the driven side.
- Sometimes in a belt-drive, there is always a possibility of some slipping taking place between the belt and the pulleys which cause the drive pulley to rotate at a lesser speed.
- Consequently reduces the power transmission. Hence belt drives are said to be not a positive type of power transmission system.

There are four commonly used types of belt are:

Flat belt

V belt

Circular

Flat belt :

This belt has a rectangular cross-section. These belts are capable of transmitting power over long distances between pulley centres.

The efficiency of this drive is around 98% and produce little ~~not~~ noise.

V-belt :

V-belt also used with grooved pulley. V-belts are trapezoidal in cross-section.

These belts permit large speed ratios and can transmit higher power. Multiple drives are possible.

Circular :

This type of belt has a circular

cross-section and is used with the grooved pulleys.

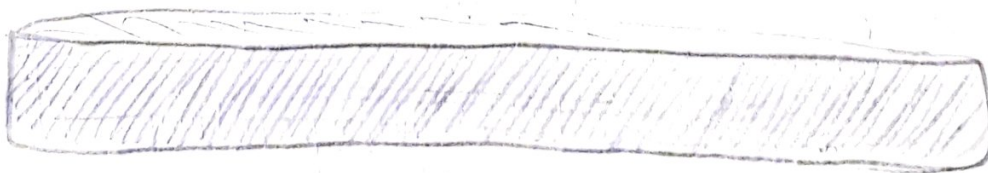
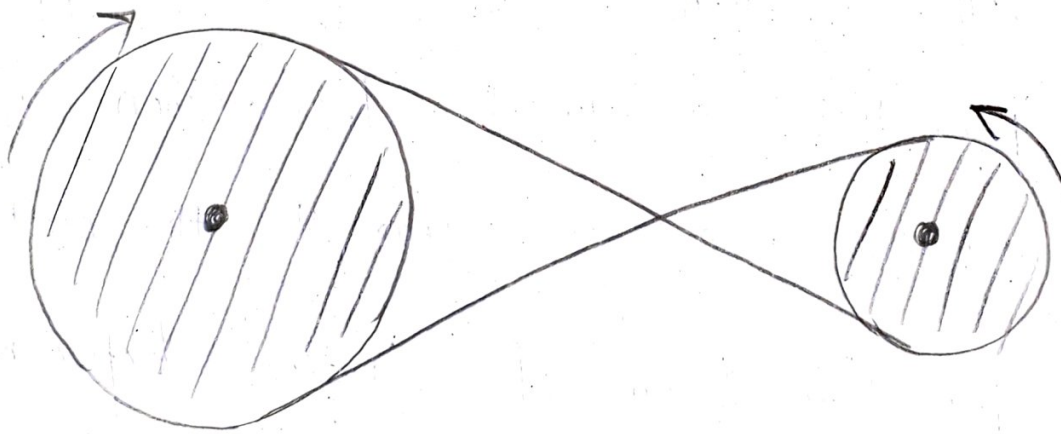
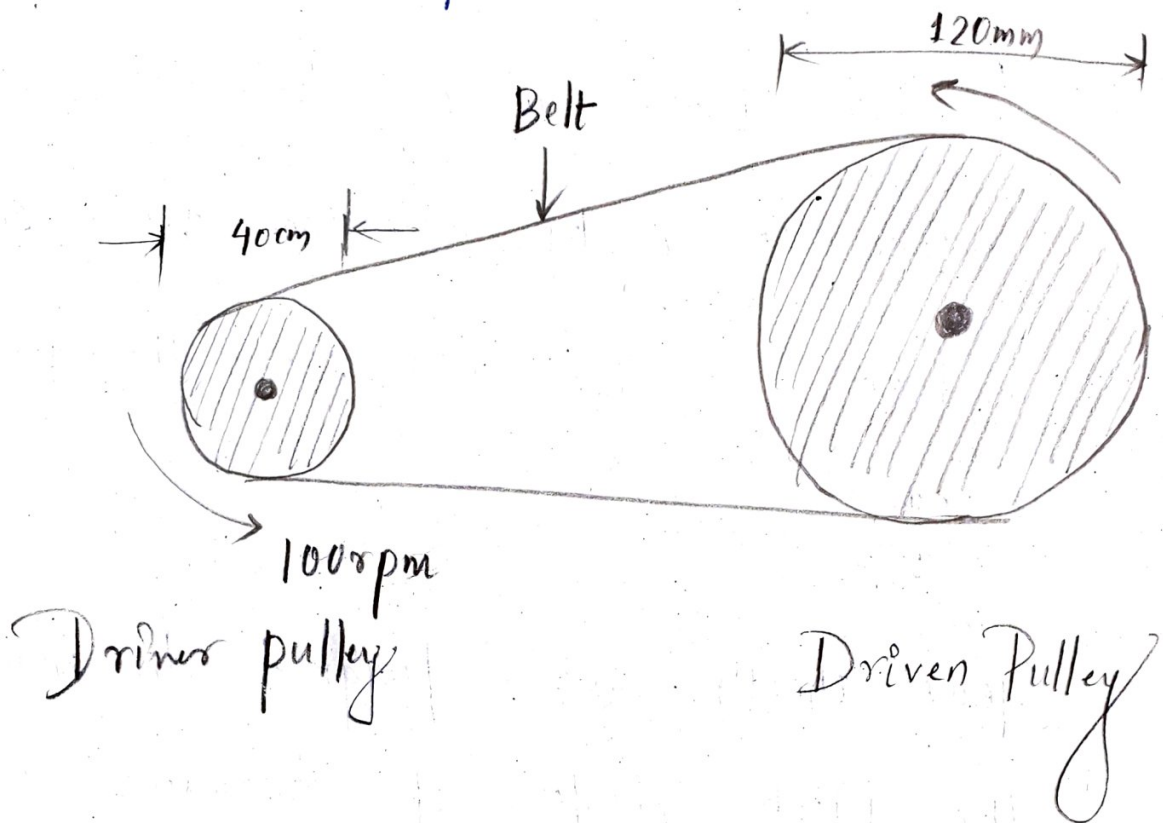
Slip:

- Consider an open belt drive rotating in a clockwise direction, this rotating of the belt over the pulleys is assumed to be due to firm frictional grip between the belt and pulleys.
- When this frictional grip becomes insufficient, there is a possibility of forwarding motion of driver without pulley with it, this is known as the slip in a belt.
- Therefore slip may be defined as the relative motion between the pulley and the belt in it. This reduces the velocity ratio and usually expressed in % and it is denoted by S .

CREEP

- Consider as an open belt drive rotating in the clockwise direction.
- The portion of the belt leaving the driver and entering the driven is known as the tight side and a portion of the belt leaving the driven and entering the driver is known as the slack side.
- During rotation, there is an expansion of a belt on a tight side and contraction of the belt on the slack side.
- Due to this uneven expansion or contraction of the belt over the pulleys, there will be a relative movement (motion) of the belt.

over the pulleys this phenomenon is known as Creep in a belt.



Flat Belt

GEAR TRAIN

A gear train is a mechanical system formed by mounting gears on a frame so the teeth of the gears engage.

Gear teeth are designed to ensure the pitch circles of engaging gears roll on each other without slipping, providing a smooth transmission of rotation from one gear to the next.

The transmission of rotation between contracting toothed wheels can be traced back to the Antikythera mechanism of Greece and south-pointing chariot of China. Illustrations by the Renaissance

scientist Georgius Agricola show gears trains with cylindrical teeth. The implementation of the involute tooth yielded a standard gear design that provides a constant speed ratio.

Features of gears and gears trains include:

The ratio of the pitch circles of mating gears defines the speed ratio and the mechanical advantages of gear set.

A planetary gear train provides high gear reduction in compact package.

It is possible to design gear teeth for gears that are non-circular, yet still transmit torque smoothly.

The speed ratios of chain and belt

drives are computed in the same way as gear ratios.

Gear teeth are distributed along the circumference of the pitch circle so the thickness t of each tooth and the space between neighbouring teeth are same. The pitch p of the gear, which is the distance between equivalent points on neighbouring teeth along the pitch circle, is equal to twice the thickness.

Speed ratio

Gear teeth are distributed along the circumference of the pitch circle so the thickness t of each tooth and the space between neighbouring teeth

are the same.

The pitch p of the gear, which is the distance between ~~equit~~ equivalent points on neighbouring teeth along the pitch circle, is equal to twice the thickness of a tooth,

Gear Drive / Toothed wheels

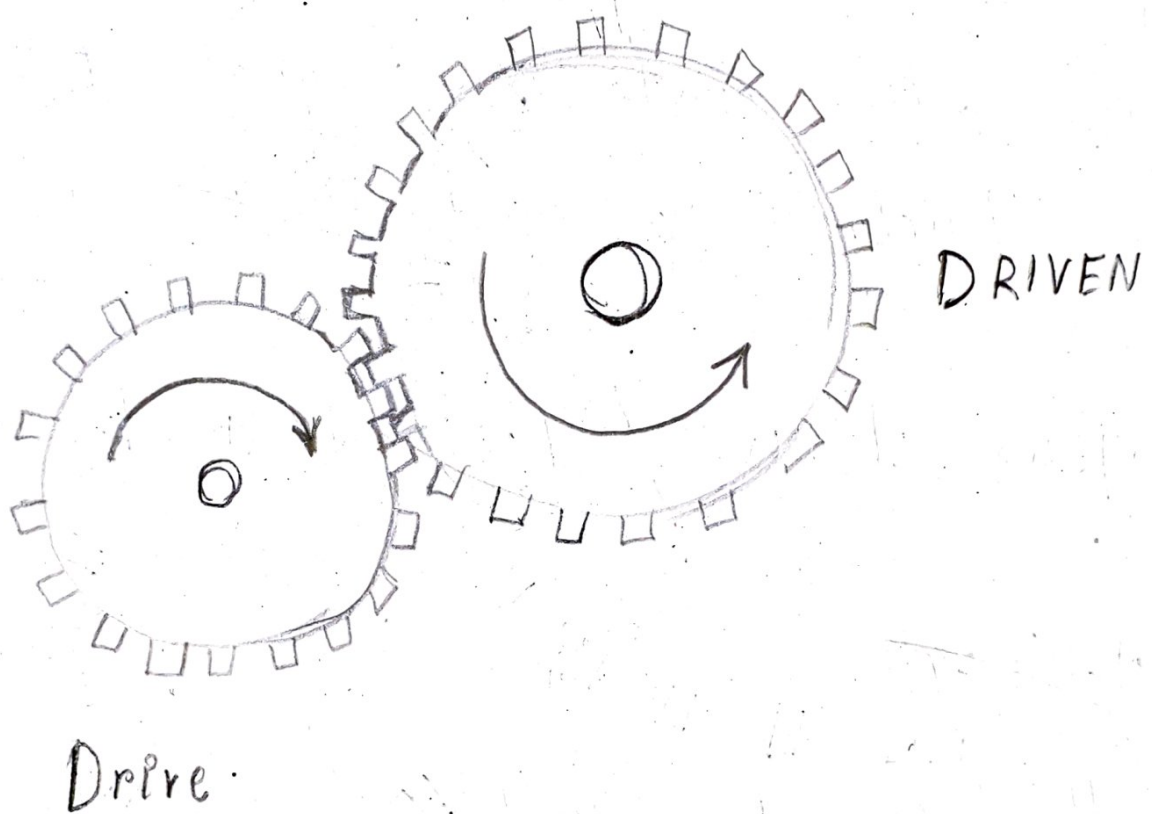
Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power
3. It has high efficiency.
4. It has reliable service
5. It has compact layout.

Disadvantages

1. The manufacture of gears requires special tools and equipment.

2. The error in cutting may cause vibrations and noise during operations.



Types

Gear Trains

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels.

Types of Gear Trains

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train

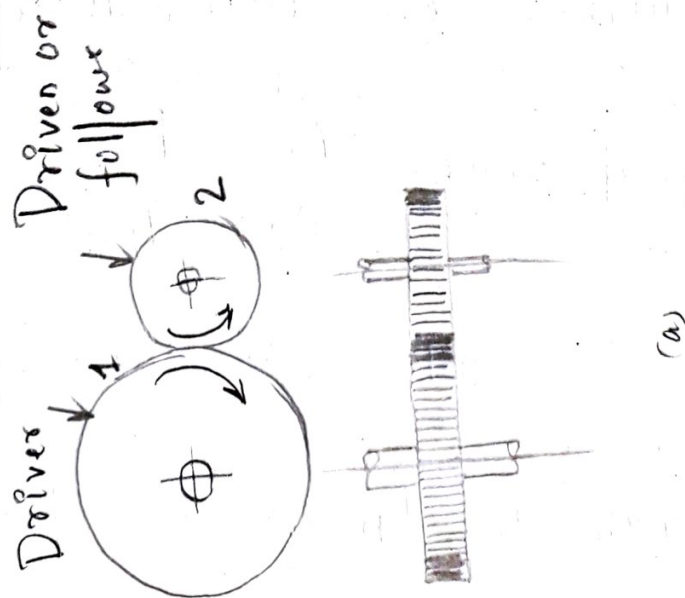
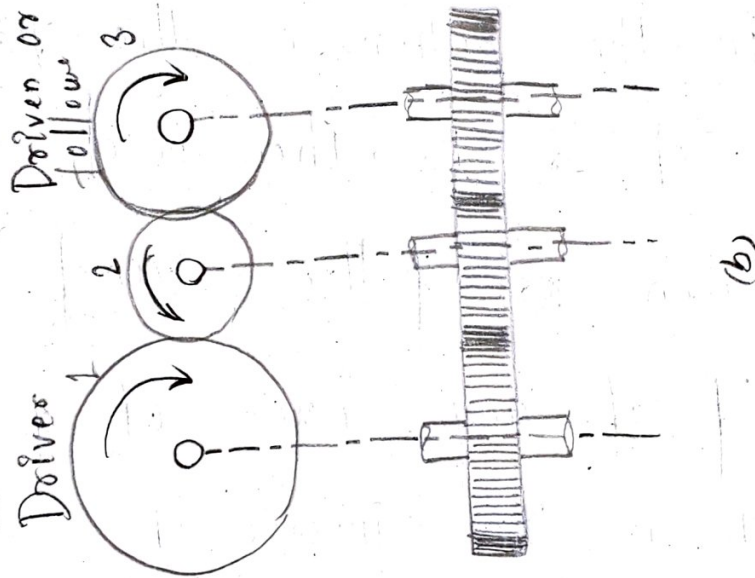
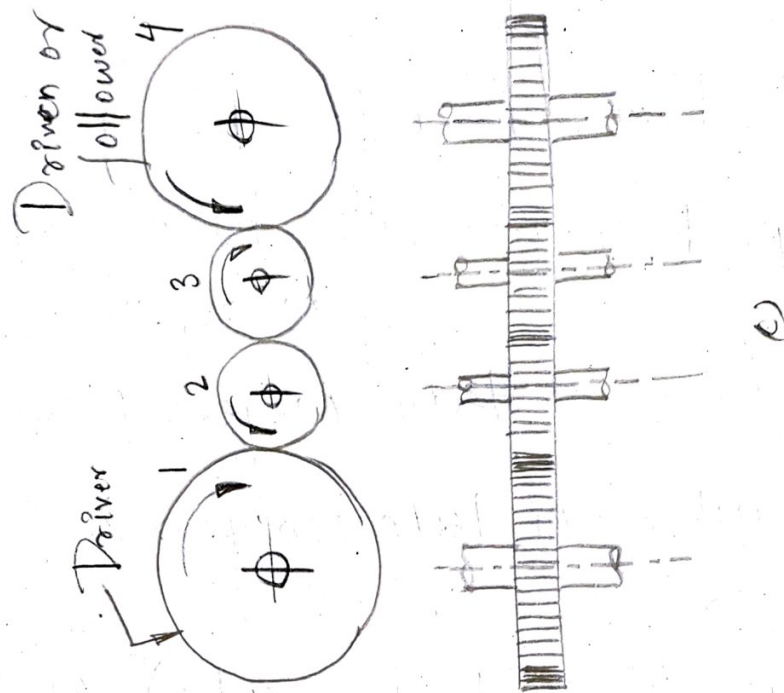
Simple Gear Train

When there is one gear on each shaft, as shown in figure it is known as simple gear train. The gears are

represented by their pitch circles.

When the distance between two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in figure.

Since the gear 1 drives the gear 2, therefore gear 1 is called the driver and the gear 2 is called the driven or follower. It may be noted that the motion of the driven gears is opposite to the motion of driving gear.



N_1 = Speed of gear 1 (or driver) in r.p.m.
 N_2 = Speed of gear 2 (or driven or follower) in r.p.m.

T_1 = Number of teeth on gear 1, and

T_2 = Number of teeth on gear 2.

Speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speed of any pair of gears in mesh is the inverse of their number of teeth, therefore.

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that the speed of the driver is known as train value

of the gear train.

Mathematically,

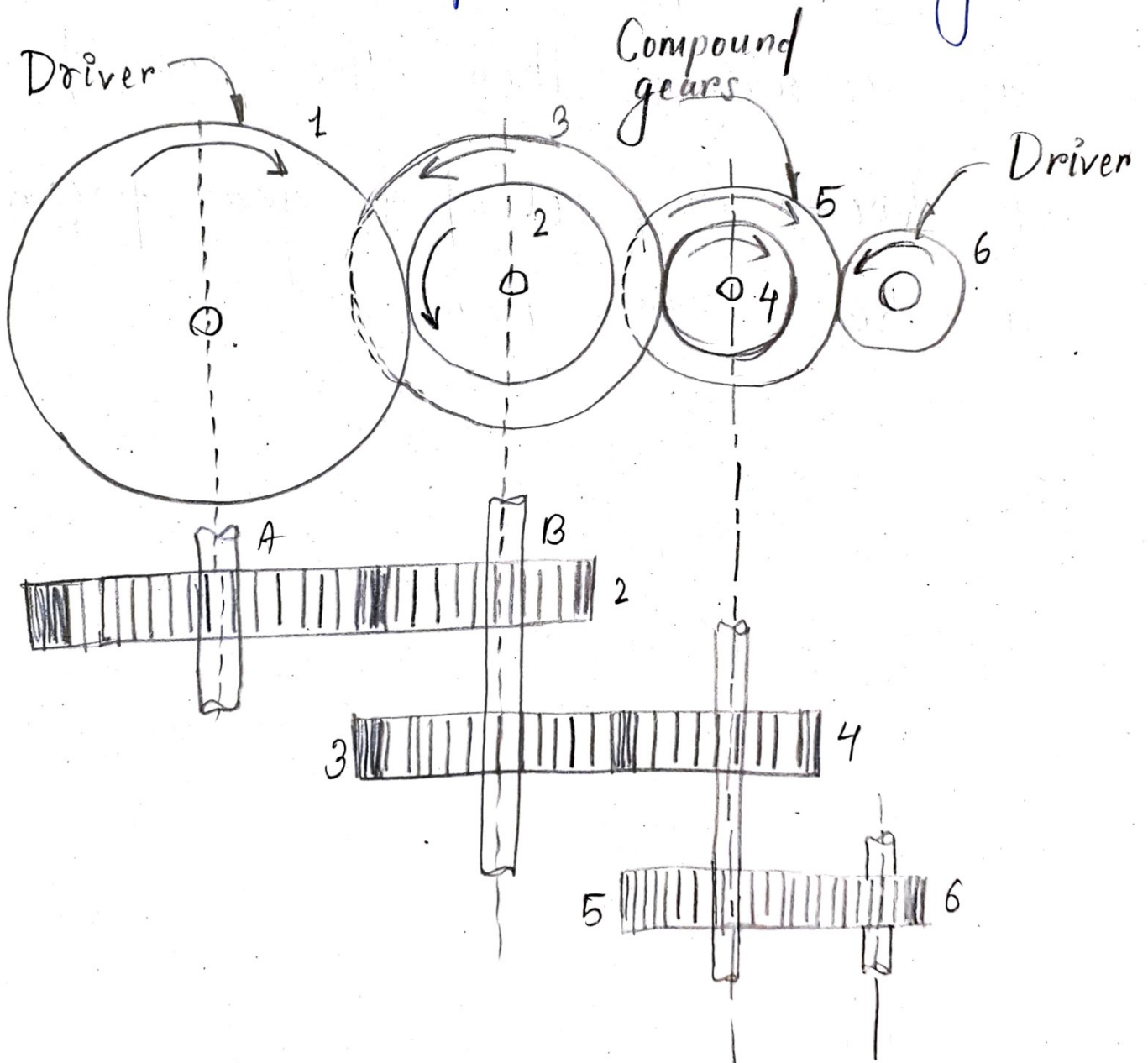
$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_2}{T_1}$$

From above, we see that train value is the reciprocal of speed ratio.

Some times the distance between the two gears is large. The motion from one gear to another in such cases

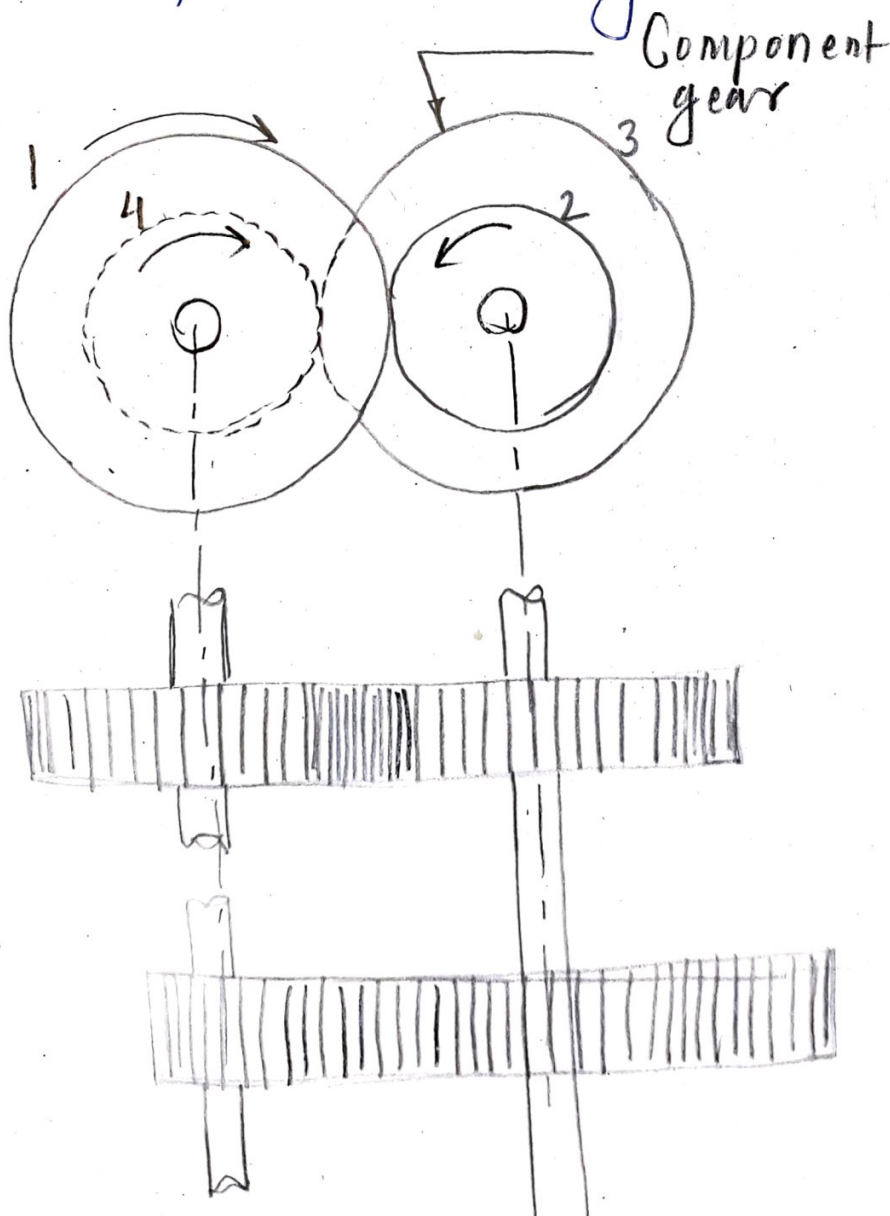
Compound Gear Train

When there are more than one gear on a shaft, as shown figure, it is called a compound train of gear.



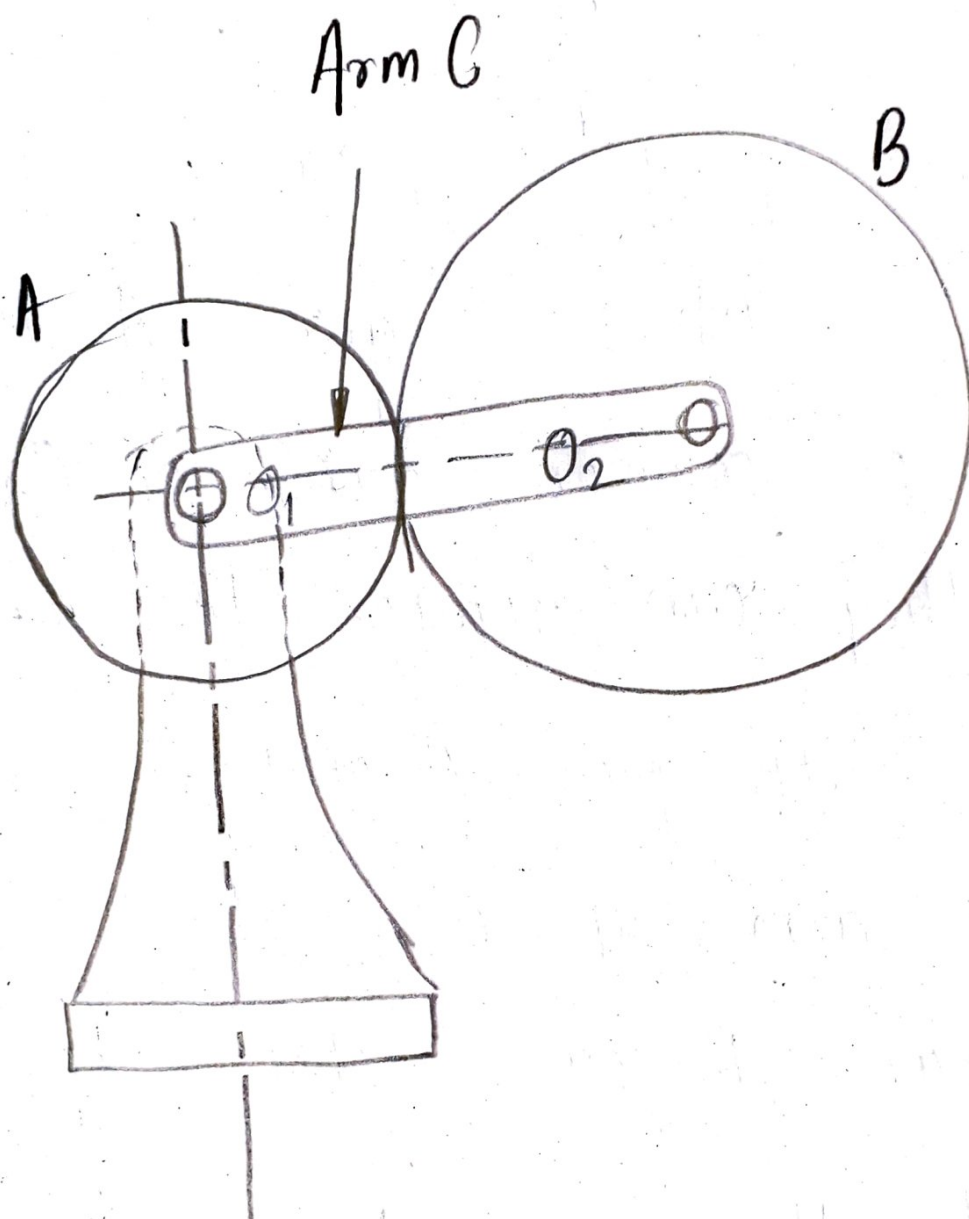
Reverted Gear Train

When the axes of the first gear (i.e. first driver) and the last gear are co-axial, then the gear train is known as reverted gear train as shown in figure.



Epicyclic Gear Train

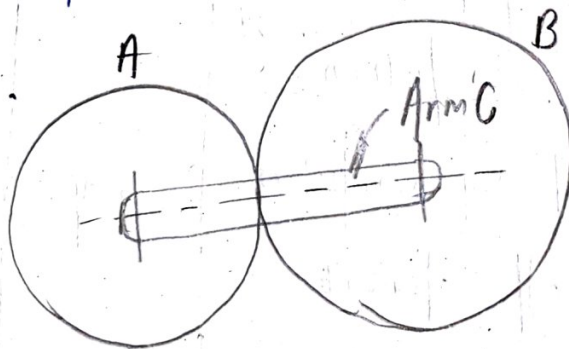
A simple epicyclic gear train is shown in figure. Where a gear A and the arm C have a common axis at O_1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2 , about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vice-versa, but if gear A is fixed and the arm is rotated about the axis of gear A (ie. O_1), the gear B is forced to rotate upon and around gear A. ~~By~~ Such motion is called epicyclic.



Epicyclic gear train

Example 13.4

In an epicyclo gear train, an arm carries two gear A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m in the anticlockwise direction, what will be the speed of gear B? If the gear A instead of being fixed makes 300 r.p.m in the clockwise direction, what will be the speed of gear B?



Given, $T_A = 36$

$$T_B = 45$$

$$N_C = 150 \text{ r.p.m (anti-clockwise)} \therefore$$

The gear train is shown in figure.

Example 13.4

Tabular Method

Sl. No.	Calculation	Revolution of element		
		Arm C	Gears A	Gears B
1.	Arm fixed-gears A rotates through +1 revolution (i.e. 1 rev. anti-clockwise)	0	+1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through +x revolution	0	+x	$-x \times \frac{T_A}{T_B}$
3.	Add +y revolution to all elements	+y	+y	+y
4.	Total motion	+y	x+y	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 rpm anti-clockwise, therefore from the fourth

row of the table,

$$y = \pm 150 \text{ r.p.m}$$

Also the gear A is fixed, therefore

$$x + y = 0. \quad \text{or} \quad x = -y = -150 \text{ rpm}$$

∴ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +270 \text{ rpm}$$

(anti-clockwise)

Speed of gear B when gear A makes
300 rpm clockwise

Since the gear A makes 300 rpm
clockwise, therefore from the fourth
row of the table,

$$x + y = -300, \quad \text{or}$$

$$x = -300 - y = -300 - 150 = -450 \text{ rpm}$$

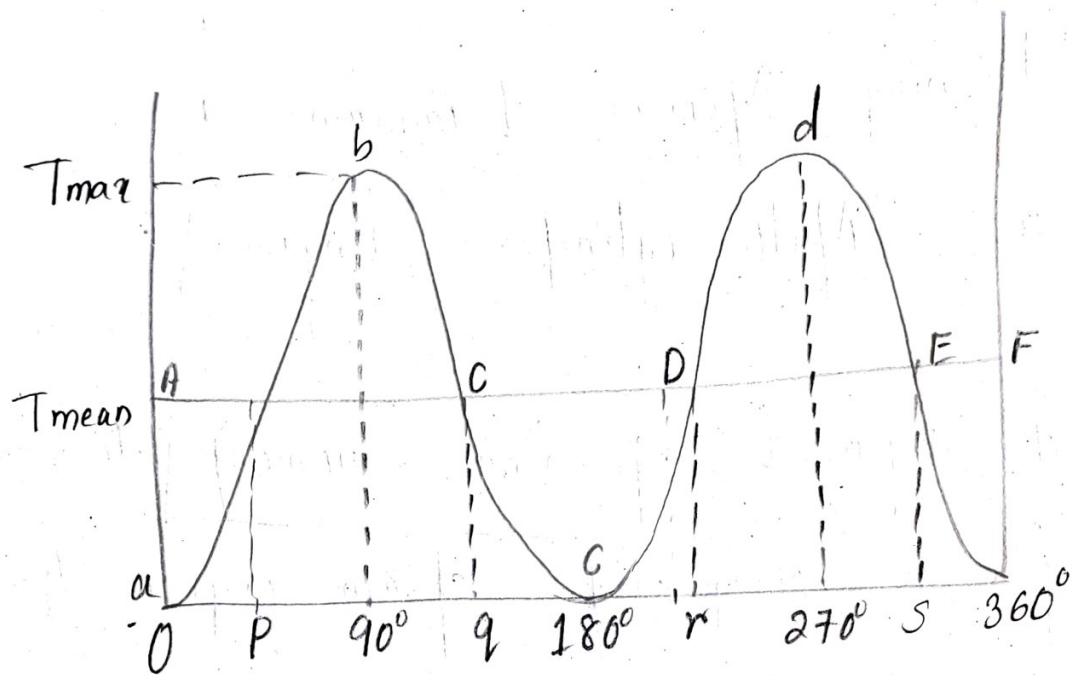
∴ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B}$$

$$= 150 + 450 \times \frac{36}{45}$$

$$= +510 \text{ r.p.m}$$

$$= 510 \text{ r.p.m (anti-clockwise)}$$

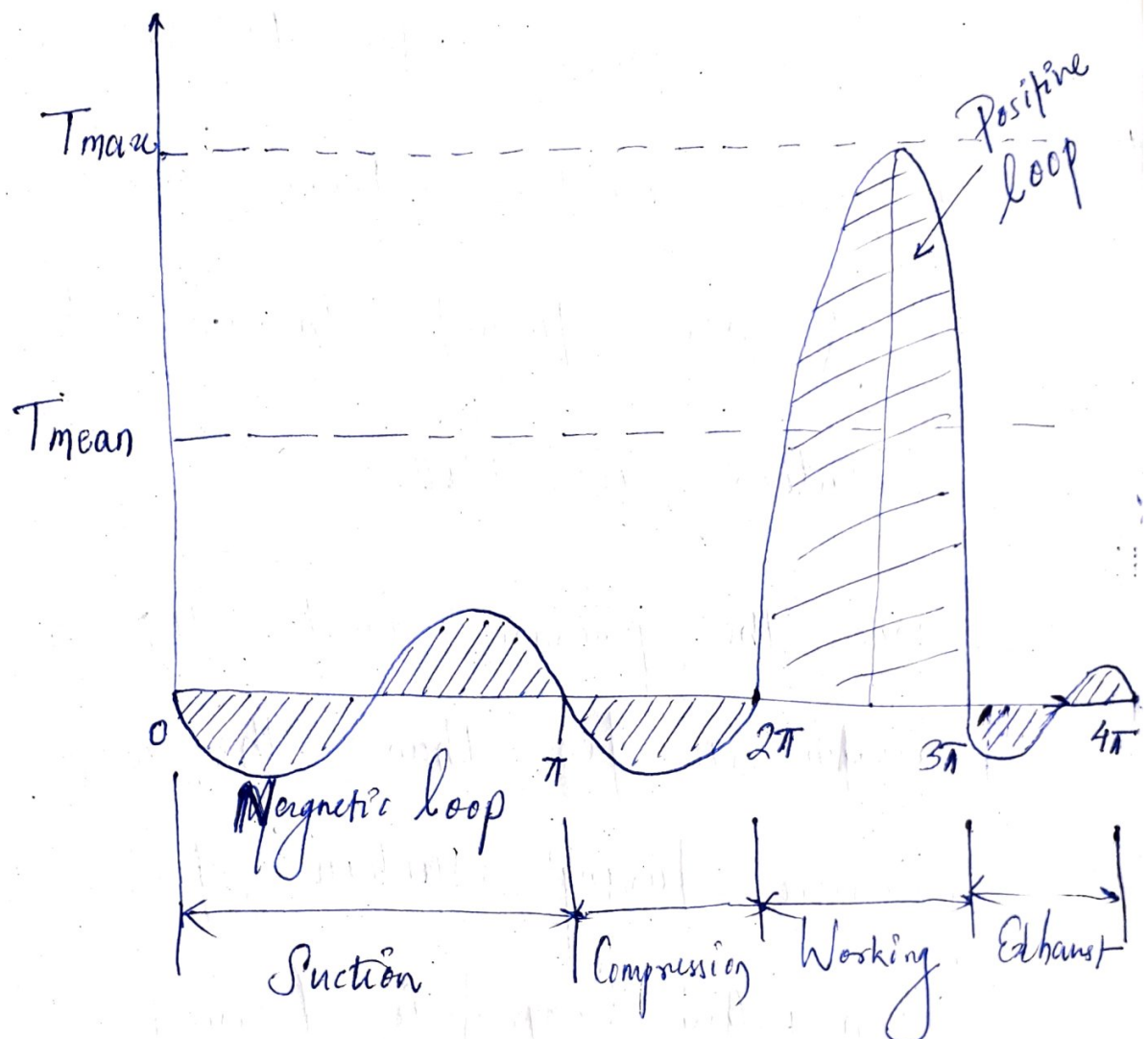


From the above expression, we see that the turning moment (T) is zero; when the crank angle (θ) is zero. It is maximum when the crank angle is 90° . and it is again zero when crank angle is 180° .

This is shown by the curve abc in and it represents the turning moment diagram for ~~instroke~~ an out stroke.

The curve cde is the turning moment diagram for instroke and is somewhat similar to the curve abc.

Turning moment diagram for 4-stroke cycle



$0 - 4\pi$ Complete stroke

$0 - \pi$ ~~$\pi - 2\pi$~~ → Suction stroke (-)

$\pi - 2\pi$ - Compression stroke (-)

$2\pi - 3\pi$ - Working stroke (+)

$3\pi - 4\pi$ - Exhaust stroke (-)

A turning moment diagram for a four stroke cycle IC engine, we know that in a four stroke cycle IC engine, there is one working stroke after a crank has turned through two revolution i.e. 720°.

Since the pressure inside the engine cylinder is less than the atmospheric pressure during suction stroke therefore a negative loop is formed.

During the compression stroke, the work is done on gases, therefore a higher negative loop is obtained.

During ~~working~~ the expansion or working stroke, the fuel burns and the gases expand, therefore a positive loop is obtained. In this stroke the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore negative loop is formed.

It may be noted that effect of inertia forces on the piston is taken into account.

Turning Moment Diagram for a Multi-cylinder Engine

A separate turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown figure.

The resultant turning moment diagram is the sum of the turning moment diagrams for three cylinders.

It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate

cylinder and third cylinder is the low pressure cylinder.
The crank, in case of three cylinders, are usually placed at 120° to each other.

